

Real 4D Lie algebras

J. Patera, P. Winternitz. Subalgebras of real three and four--dimensional Lie algebras.

Journal of Mathematical Physics, 18, 1449--1455, 1977.

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In[1]:= SetDirectory[NotebookDirectory[]];
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In[2]:= << "Symbolie.wl"
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Symbolie (v. 1.6) - A Package for determining Optimal Systems of Lie Subalgebras.

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In[3]:= CS = Table[0, {k, 1, 4}, {i, 1, 4}, {j, 1, 4}];
AllCS = Table[CS, {r, 1, 30}];
AllCS[[2, 2, 1, 2]] = 1; AllCS[[2, 2, 2, 1]] = -1;
AllCS[[3, 2, 1, 2]] = 1; AllCS[[3, 2, 2, 1]] = -1;
AllCS[[3, 4, 3, 4]] = 1; AllCS[[3, 4, 4, 3]] = -1;
AllCS[[4, 1, 2, 3]] = 1; AllCS[[4, 1, 3, 2]] = -1;
AllCS[[5, 1, 1, 3]] = 1; AllCS[[5, 1, 3, 1]] = -1;
AllCS[[5, 1, 2, 3]] = 1; AllCS[[5, 1, 3, 2]] = -1;
AllCS[[5, 2, 2, 3]] = 1; AllCS[[5, 2, 3, 2]] = -1;
AllCS[[6, 1, 1, 3]] = 1; AllCS[[6, 1, 3, 1]] = -1;
AllCS[[6, 2, 2, 3]] = 1; AllCS[[6, 2, 3, 2]] = -1;
AllCS[[7, 1, 1, 3]] = 1; AllCS[[7, 1, 3, 1]] = -1;
AllCS[[7, 2, 2, 3]] = -1; AllCS[[7, 2, 3, 2]] = 1;
AllCS[[8, 1, 1, 3]] = 1; AllCS[[8, 1, 3, 1]] = -1;
AllCS[[8, 2, 2, 3]] = a;
AllCS[[8, 2, 3, 2]] = -a;
(*con abs(a) tra 0 e 1*)AllCS[[9, 2, 1, 3]] = -1;
AllCS[[9, 2, 3, 1]] = 1;
AllCS[[9, 1, 2, 3]] = 1; AllCS[[9, 1, 3, 2]] = -1;
AllCS[[10, 1, 1, 3]] = a;
AllCS[[10, 1, 3, 1]] = -a;
(*a>0*)AllCS[[10, 1, 2, 3]] = 1;
AllCS[[10, 1, 3, 2]] = -1;
AllCS[[10, 2, 1, 3]] = -1; AllCS[[10, 2, 3, 1]] = 1;
AllCS[[10, 2, 2, 3]] = a;
AllCS[[10, 2, 3, 2]] = -a;
(*a>0*)AllCS[[11, 2, 3, 1]] = 2;
AllCS[[11, 2, 1, 3]] = -2;
AllCS[[11, 1, 1, 2]] = 1; AllCS[[11, 1, 2, 1]] = -1;
AllCS[[11, 3, 2, 3]] = 1; AllCS[[11, 3, 3, 2]] = -1;
AllCS[[12, 3, 1, 2]] = 1; AllCS[[12, 3, 2, 1]] = -1;
AllCS[[12, 1, 2, 3]] = 1; AllCS[[12, 1, 3, 2]] = -1;
AllCS[[12, 2, 3, 1]] = 1; AllCS[[12, 2, 1, 3]] = -1;
AllCS[[13, 1, 2, 4]] = 1; AllCS[[13, 1, 4, 2]] = -1;
AllCS[[13, 2, 3, 4]] = 1; AllCS[[13, 2, 4, 3]] = -1;
AllCS[[14, 1, 1, 4]] = a;
AllCS[[14, 1, 4, 1]] = -a;
(*a diverso da 0,1*)AllCS[[14, 2, 2, 4]] = 1;
AllCS[[14, 2, 4, 2]] = -1;
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AllCS[[14, 2, 3, 4]] = 1; AllCS[[14, 2, 4, 3]] = -1;
AllCS[[14, 3, 3, 4]] = 1; AllCS[[14, 3, 4, 3]] = -1;
AllCS[[15, 1, 1, 4]] = 1; AllCS[[15, 1, 4, 1]] = -1;
AllCS[[15, 2, 2, 4]] = 1; AllCS[[15, 2, 4, 2]] = -1;
AllCS[[15, 2, 3, 4]] = 1; AllCS[[15, 2, 4, 3]] = -1;
AllCS[[15, 3, 3, 4]] = 1; AllCS[[15, 3, 4, 3]] = -1;
AllCS[[16, 1, 1, 4]] = 1; AllCS[[16, 1, 4, 1]] = -1;
AllCS[[16, 2, 3, 4]] = 1; AllCS[[16, 2, 4, 3]] = -1;
AllCS[[17, 1, 1, 4]] = 1; AllCS[[17, 1, 4, 1]] = -1;
AllCS[[17, 1, 2, 4]] = 1; AllCS[[17, 1, 4, 2]] = -1;
AllCS[[17, 2, 2, 4]] = 1; AllCS[[17, 2, 4, 2]] = -1;
AllCS[[17, 2, 3, 4]] = 1; AllCS[[17, 2, 4, 3]] = -1;
AllCS[[17, 3, 3, 4]] = 1; AllCS[[17, 3, 4, 3]] = -1;
AllCS[[18, 1, 1, 4]] = 1; AllCS[[18, 1, 4, 1]] = -1;
AllCS[[18, 2, 2, 4]] = a; AllCS[[18, 2, 4, 2]] = -a;
AllCS[[18, 3, 3, 4]] = b;
AllCS[[18, 3, 4, 3]] = -b;
(*-1<a,b<1,ab≠0*)AllCS[[19, 1, 1, 4]] = 1;
AllCS[[19, 1, 4, 1]] = -1;
AllCS[[19, 2, 2, 4]] = a; AllCS[[19, 2, 4, 2]] = -a;
AllCS[[19, 3, 3, 4]] = a;
AllCS[[19, 3, 4, 3]] = -a;
(*-1<a<1,a≠0*)AllCS[[20, 1, 1, 4]] = 1;
AllCS[[20, 1, 4, 1]] = -1;
AllCS[[20, 2, 2, 4]] = a;
AllCS[[20, 2, 4, 2]] = -a;
(*-1<a<1,a≠0*)AllCS[[20, 3, 3, 4]] = 1;
AllCS[[20, 3, 4, 3]] = -1;
AllCS[[21, 1, 1, 4]] = 1; AllCS[[21, 1, 4, 1]] = -1;
AllCS[[21, 2, 2, 4]] = 1; AllCS[[21, 2, 4, 2]] = -1;
AllCS[[21, 3, 3, 4]] = 1; AllCS[[21, 3, 4, 3]] = -1;
AllCS[[22, 1, 1, 4]] = a;
AllCS[[22, 1, 4, 1]] = -a;
(*a≠0,b≥0*)AllCS[[22, 2, 2, 4]] = b;
AllCS[[22, 2, 4, 2]] = -b;
AllCS[[22, 2, 3, 4]] = 1; AllCS[[22, 2, 4, 3]] = -1;
AllCS[[22, 3, 2, 4]] = -1; AllCS[[22, 3, 4, 2]] = 1;
AllCS[[22, 3, 3, 4]] = b; AllCS[[22, 3, 4, 3]] = -b;
AllCS[[23, 1, 1, 4]] = 2; AllCS[[23, 1, 4, 1]] = -2;
AllCS[[23, 2, 2, 4]] = 1; AllCS[[23, 2, 4, 2]] = -1;
AllCS[[23, 2, 3, 4]] = 1; AllCS[[23, 2, 4, 3]] = -1;
AllCS[[23, 3, 3, 4]] = 1; AllCS[[23, 3, 4, 3]] = -1;
AllCS[[23, 1, 2, 3]] = 1; AllCS[[23, 1, 3, 2]] = -1;
AllCS[[24, 1, 2, 3]] = 1; AllCS[[24, 1, 3, 2]] = -1;
AllCS[[24, 2, 2, 4]] = 1; AllCS[[24, 2, 4, 2]] = -1;
AllCS[[24, 3, 3, 4]] = -1; AllCS[[24, 3, 4, 3]] = 1;
AllCS[[25, 1, 2, 3]] = 1;

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AllCS[[25, 1, 3, 2]] = -1;
(*0<abs(b)<1*)AllCS[[25, 1, 1, 4]] = 1 + b;
AllCS[[25, 1, 4, 1]] = -1 - b;
AllCS[[25, 2, 2, 4]] = 1; AllCS[[25, 2, 4, 2]] = -1;
AllCS[[25, 3, 3, 4]] = b; AllCS[[25, 3, 4, 3]] = -b;
AllCS[[26, 1, 2, 3]] = 1; AllCS[[26, 1, 3, 2]] = -1;
AllCS[[26, 1, 1, 4]] = 2; AllCS[[26, 1, 4, 1]] = -2;
AllCS[[26, 2, 2, 4]] = 1; AllCS[[26, 2, 4, 2]] = -1;
AllCS[[26, 3, 3, 4]] = 1; AllCS[[26, 3, 4, 3]] = -1;
AllCS[[27, 1, 2, 3]] = 1; AllCS[[27, 1, 3, 2]] = -1;
AllCS[[27, 1, 1, 4]] = 1; AllCS[[27, 1, 4, 1]] = -1;
AllCS[[27, 2, 2, 4]] = 1; AllCS[[27, 2, 4, 2]] = -1;
AllCS[[28, 1, 2, 3]] = 1; AllCS[[28, 1, 3, 2]] = -1;
AllCS[[28, 3, 2, 4]] = -1; AllCS[[28, 3, 4, 2]] = 1;
AllCS[[28, 2, 3, 4]] = 1; AllCS[[28, 2, 4, 3]] = -1;
AllCS[[29, 1, 2, 3]] = 1;
AllCS[[29, 1, 3, 2]] = -1;
(*a>0*)AllCS[[29, 1, 1, 4]] = 2 a;
AllCS[[29, 1, 4, 1]] = -2 a;
AllCS[[29, 2, 2, 4]] = a; AllCS[[29, 2, 4, 2]] = -a;
AllCS[[29, 3, 2, 4]] = -1; AllCS[[29, 3, 4, 2]] = 1;
AllCS[[29, 2, 3, 4]] = 1; AllCS[[29, 2, 4, 3]] = -1;
AllCS[[29, 3, 3, 4]] = a; AllCS[[29, 3, 4, 3]] = -a;
AllCS[[30, 1, 1, 3]] = 1; AllCS[[30, 1, 3, 1]] = -1;
AllCS[[30, 2, 2, 3]] = 1; AllCS[[30, 2, 3, 2]] = -1;
AllCS[[30, 2, 1, 4]] = -1; AllCS[[30, 2, 4, 1]] = 1;
AllCS[[30, 1, 2, 4]] = 1;
AllCS[[30, 1, 4, 2]] = -1;

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In[91]:= pars = Table[{{}}, {{}}, {k1, 1, 30}];
pars[[8]] = {{a}, {-1 < a, a < 1, a ≠ 0}}; (*pars[[8]]={a=1/2};*)
pars[[10]] = {{a}, {a > 0}};
pars[[14]] = {{a}, {a ≠ 0, a ≠ 1}};
pars[[18]] = {{a, b}, {-1 ≤ a, a < b, b < 1, a b ≠ 0}};
pars[[19]] = {{a}, {-1 ≤ a, a < 1, a ≠ 0}};
pars[[20]] = {{a}, {-1 ≤ a, a < 1, a ≠ 0}};
pars[[22]] = {{a, b}, {a ≠ 0, b ≥ 0}};
pars[[25]] = {{b}, {-1 < b, b < 1, b ≠ 0}};
pars[[29]] = {{a}, {a > 0}};

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In[93]:= allalg1 = {}; allalg2 = {}; allalg3 = {};

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In[94]:= For[k1 = 1, k1 ≤ 30, k1++,
  Print["====="];
  Print["Algebra n. ", k1];
  t1 = SessionTime[];
  Print[CommutatorTable[AllCS[[k1]] // MatrixForm];
  alg1 = SubAlgebra[AllCS[[k1]], pars[[k1]], 1];
  AppendTo[allalg1, alg1];
  optalg1 = PrintOptimal[alg1];
  Print[optalg1];
  G = PrintGraph[alg1];
  Print[G];
  alg2 = SubAlgebra[AllCS[[k1]], pars[[k1]], 2];
  AppendTo[allalg2, alg2];
  optalg2 = PrintOptimal[alg2];
  Print[optalg2];
  G = PrintGraph[alg2];
  Print[G];
  alg3 = SubAlgebra[AllCS[[k1]], pars[[k1]], 3];
  AppendTo[allalg3, alg3];
  optalg3 = PrintOptimal[alg3];
  Print[optalg3];
  G = PrintGraph[alg3];
  Print[G];
  t1 = SessionTime[] - t1;
  Print["Time of computation: ", t1]
];
Print["====="];

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Algebra n. 1

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

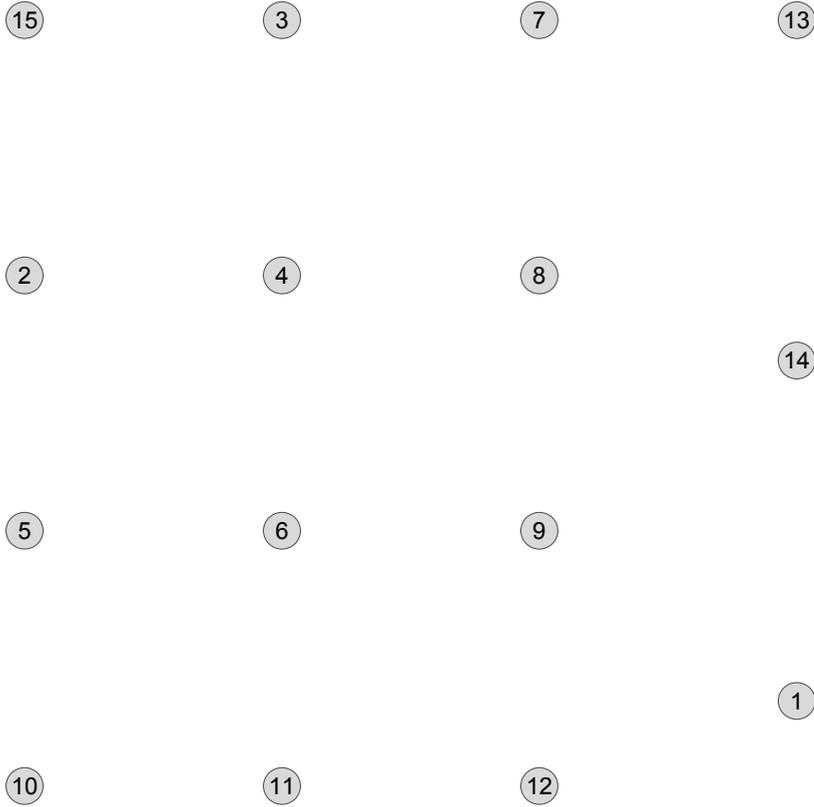
Done.

There are 15 optimal families of 1-dimensional Lie subalgebras.

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{{E1}, {E2}, {E3}, {E4}, {E1 + a1 E2}, {E1 + a1 E3}, {E2 + a1 E3},
 {E1 + a1 E4}, {E2 + a1 E4}, {E3 + a1 E4}, {E1 + a1 E2 + a2 E3}, {E1 + a1 E2 + a2 E4},
 {E1 + a1 E3 + a2 E4}, {E2 + a1 E3 + a2 E4}, {E1 + a1 E2 + a2 E3 + a3 E4}}
1 → {E1}, 2 → {E2}, 3 → {E3}, 4 → {E4}, 5 → {E1 + a1 E2},
6 → {E1 + a1 E3}, 7 → {E2 + a1 E3}, 8 → {E1 + a1 E4}, 9 → {E2 + a1 E4},
10 → {E3 + a1 E4}, 11 → {E1 + a1 E2 + a2 E3}, 12 → {E1 + a1 E2 + a2 E4},
13 → {E1 + a1 E3 + a2 E4}, 14 → {E2 + a1 E3 + a2 E4}, 15 → {E1 + a1 E2 + a2 E3 + a3 E4}

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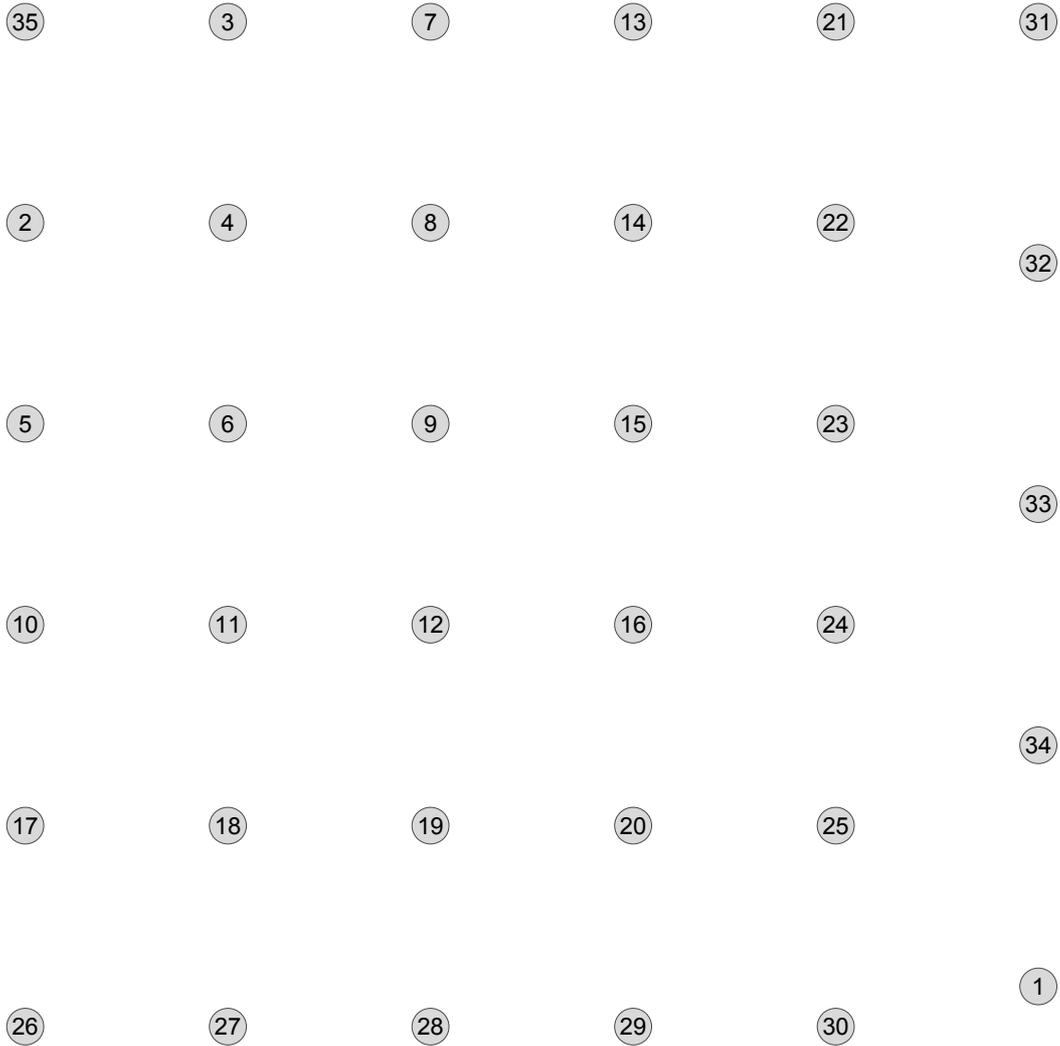


There are 35 2-D families of subalgebras to be analyzed.

Done.

There are 35 optimal families of 2-dimensional Lie subalgebras.

- $\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\},$
 $\{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\},$
 $\{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_3 + \mathfrak{a}_4 \mathfrak{E}_4\}\}$
- $1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\},$
 $6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$
 $10 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 11 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\}, 13 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\},$
 $14 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 15 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 16 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 17 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\},$
 $18 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\}, 19 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 20 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\},$
 $21 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\}, 22 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4\}, 23 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3, \mathfrak{E}_4\},$
 $24 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\}, 25 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4\}, 26 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\},$
 $27 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 28 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3\}, 29 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2\},$
 $30 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\}, 31 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\},$
 $32 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4\}, 33 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_3\},$
 $34 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_4\}, 35 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_3 \mathfrak{E}_3 + \mathfrak{a}_4 \mathfrak{E}_4\}$



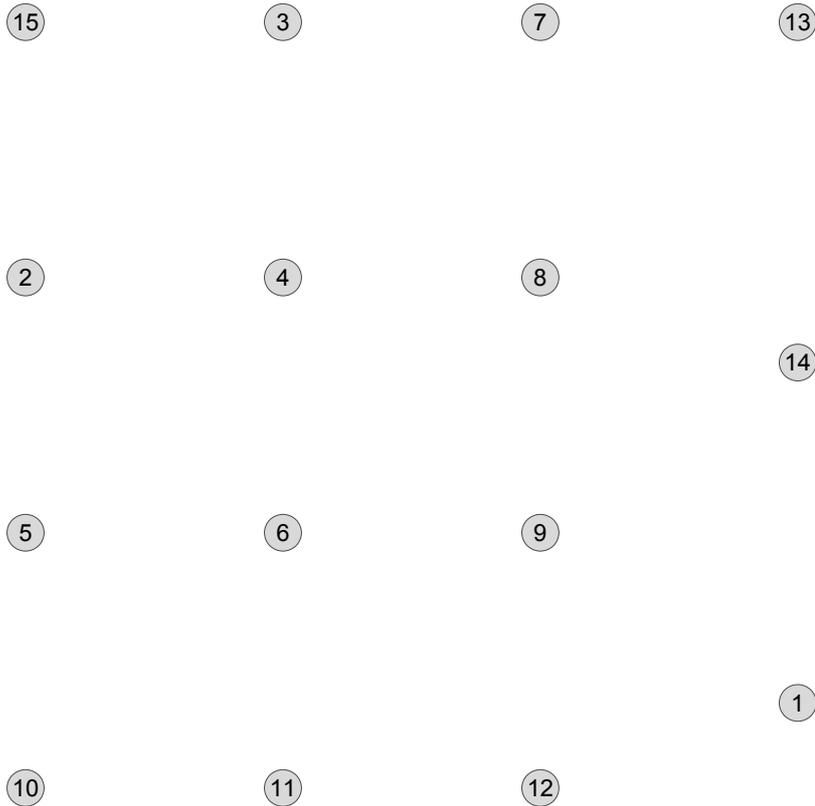
There are 15 3-D families of subalgebras to be analyzed.

Done.

There are 15 optimal families of 3-dimensional Lie subalgebras.

$$\{ \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 \}, \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4 \}, \{ \mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4 \}, \{ \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4 \}, \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4 \}, \\ \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4 \}, \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 \}, \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4 \}, \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4 \}, \\ \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 \}, \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4 \}, \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3, \mathfrak{E}_4 \}, \\ \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4 \}, \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 \}, \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4 \} \}$$

$$\{ 1 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 \}, 2 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4 \}, 3 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4 \}, 4 \rightarrow \{ \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4 \}, \\ 5 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4 \}, 6 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4 \}, 7 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 \}, \\ 8 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4 \}, 9 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4 \}, 10 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 \}, \\ 11 \rightarrow \{ \mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4 \}, 12 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3, \mathfrak{E}_4 \}, 13 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4 \}, \\ 14 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 \}, 15 \rightarrow \{ \mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_3 \mathfrak{E}_4 \} \}$$



Time of computation: 1.382925

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Algebra n. 2

$$\begin{pmatrix} 0 & \mathfrak{E}_2 & 0 & 0 \\ -\mathfrak{E}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

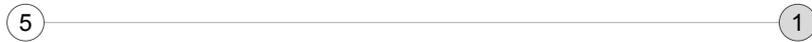
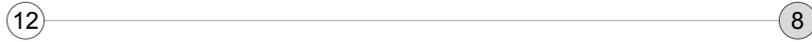
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 11 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, \\ \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 8 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, \\ 9 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 13 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 14 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



There are 23 2-D families of subalgebras to be analyzed.

Done.

There are 17 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\},$
 $\{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\},$
 $7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 11 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\},$
 $13 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 14 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 15 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\}, 16 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3\},$
 $19 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 20 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 22 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4, \mathfrak{E}_2\}\}$



There are 9 3-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$
 $7 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3\}, 9 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}\}$



Time of computation: 3.430642

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Algebra n. 3

$$\begin{pmatrix} 0 & \mathbb{E}_2 & 0 & 0 \\ -\mathbb{E}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{E}_4 \\ 0 & 0 & -\mathbb{E}_4 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

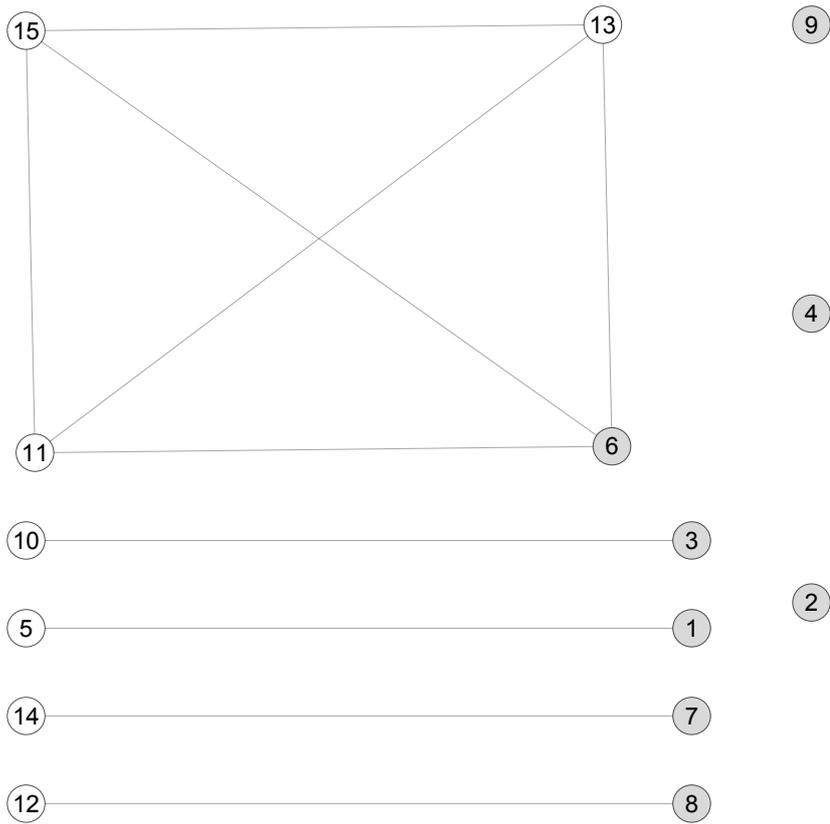
Done.

There are 8 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathbb{E}_1\}, \{\mathbb{E}_2\}, \{\mathbb{E}_3\}, \{\mathbb{E}_4\}, \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_3\}, \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_3\}, \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_4\}, \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}\}$

$\{1 \rightarrow \{\mathbb{E}_1\}, 2 \rightarrow \{\mathbb{E}_2\}, 3 \rightarrow \{\mathbb{E}_3\}, 4 \rightarrow \{\mathbb{E}_4\},$

$6 \rightarrow \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_3\}, 7 \rightarrow \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_3\}, 8 \rightarrow \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_4\}, 9 \rightarrow \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}\}$

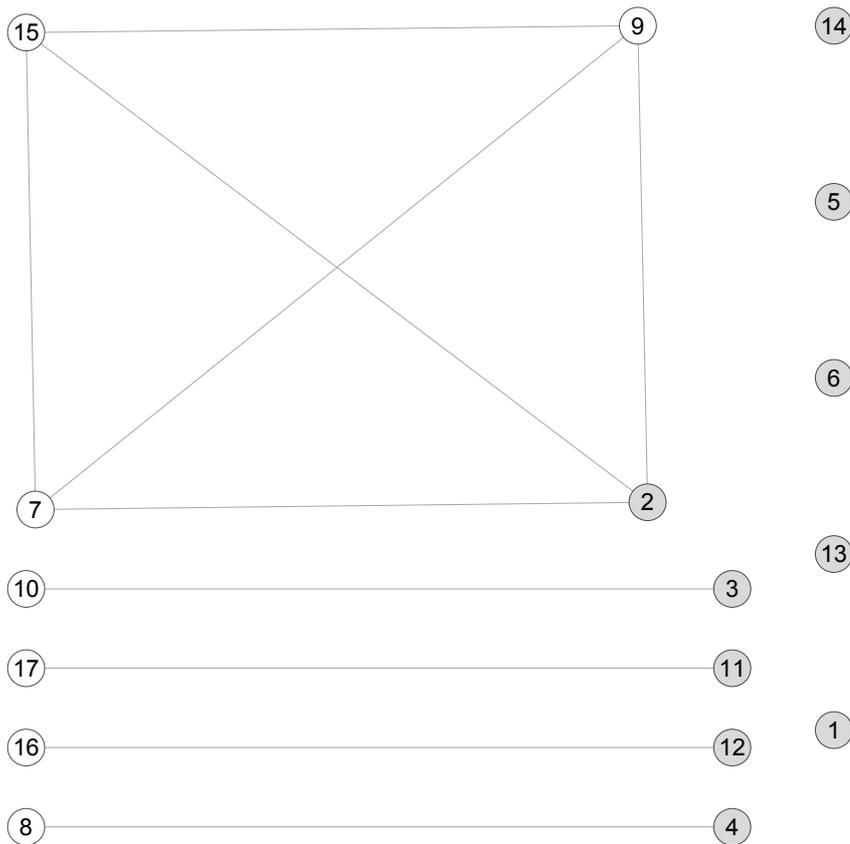


There are 17 2-D families of subalgebras to be analyzed.

Done.

There are 10 optimal families of 2-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}$
 $1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\},$
 $11 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 12 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 13 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 14 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



Time of computation: 2.441480

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Algebra n. 4

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mathfrak{a}_1 & 0 \\ 0 & -\mathfrak{a}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 9 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\},$$

$$8 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 14 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}\}$$



There are 19 2-D families of subalgebras to be analyzed.

Done.

There are 13 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\},$$

$$\{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\},$$

$$\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\},$$

$$7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 11 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2\},$$

$$13 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_3\}, 14 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 16 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_2 \mathfrak{E}_3\}\}$$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 7 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4, \mathfrak{E}_3 + a_2 \mathfrak{E}_4\}$
 $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\},$
 $5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4, \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4, \mathfrak{E}_3 + a_2 \mathfrak{E}_4\}$



Time of computation: 2.306443

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Algebra n. 5

$$\begin{pmatrix} 0 & 0 & \mathfrak{E}_1 & 0 \\ 0 & 0 & \mathfrak{E}_1 + \mathfrak{E}_2 & 0 \\ -\mathfrak{E}_1 & -\mathfrak{E}_1 - \mathfrak{E}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

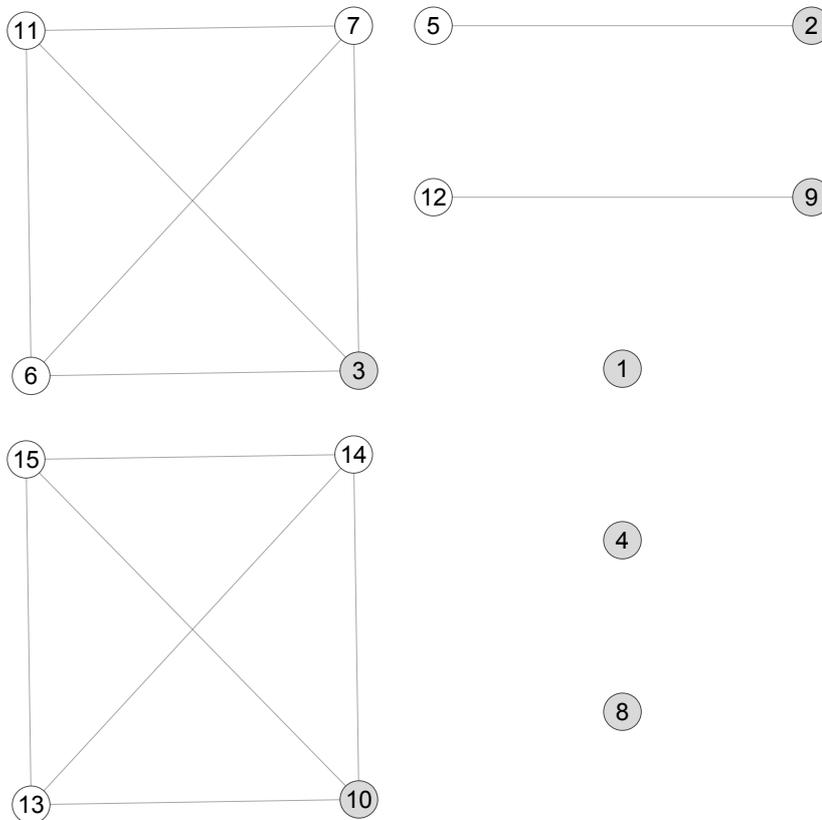
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 7 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$



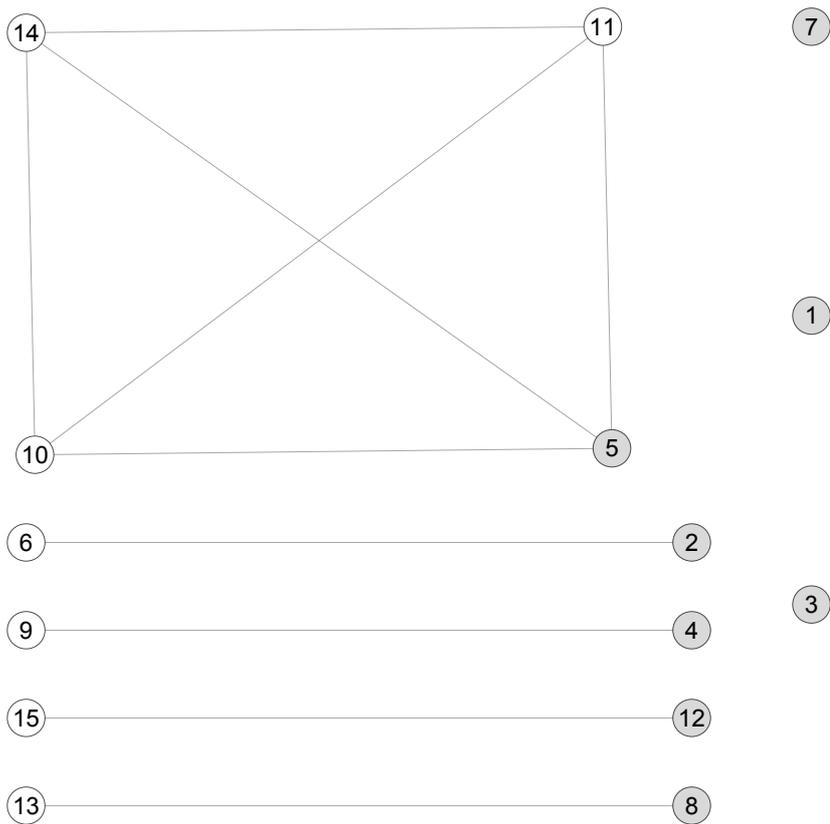
There are 15 2-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4, \mathfrak{E}_2\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4, \mathfrak{E}_2\}\}$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_3, e_4\}, \{e_1, e_2, e_3 + a_1 e_4\}\}$

$\{1 \rightarrow \{e_1, e_2, e_3\}, 2 \rightarrow \{e_1, e_2, e_4\}, 3 \rightarrow \{e_1, e_3, e_4\}, 4 \rightarrow \{e_1, e_2, e_3 + a_1 e_4\}\}$



Time of computation: 8.263238

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Algebra n. 6

$$\begin{pmatrix} 0 & 0 & \mathfrak{E}_1 & 0 \\ 0 & 0 & \mathfrak{E}_2 & 0 \\ -\mathfrak{E}_1 & -\mathfrak{E}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

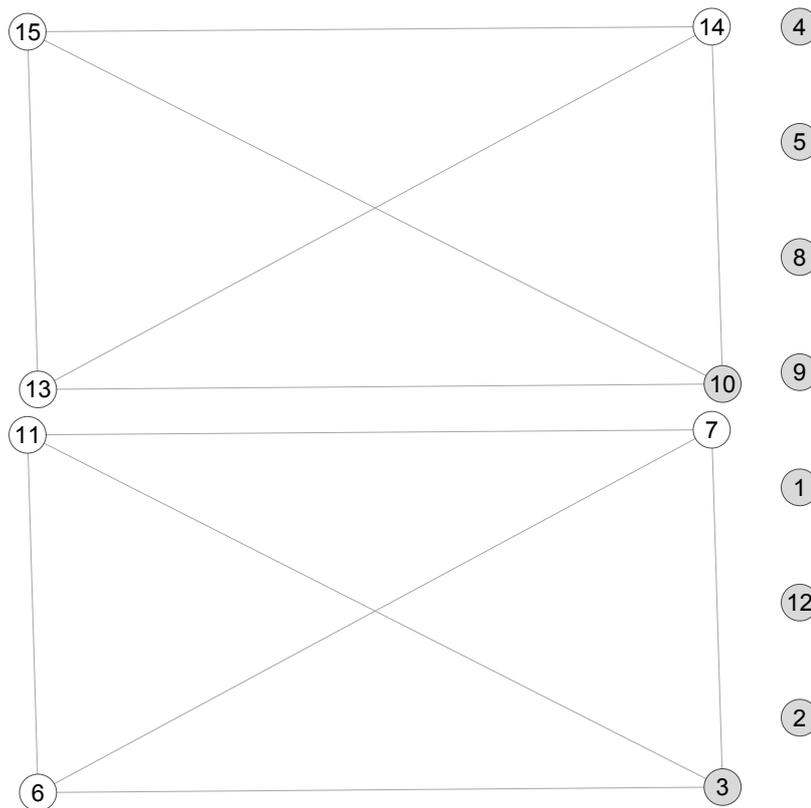
Done.

There are 9 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2\},$$

$$8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}\}$$



There are 22 2-D families of subalgebras to be analyzed.

Done.

There are 14 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\},$$

$$\{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\},$$

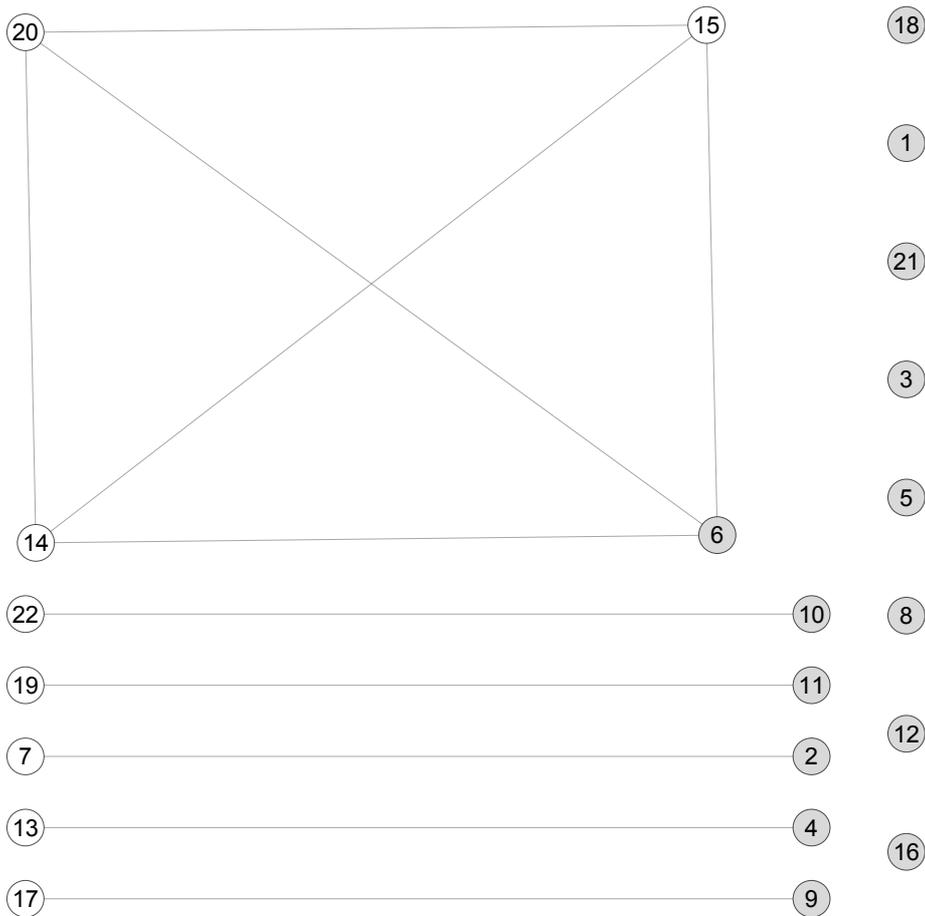
$$\{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\},$$

$$5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$$

$$10 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 11 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\},$$

$$16 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 18 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_2 \mathfrak{E}_4\}, 21 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



There are 9 3-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\},$$

$$4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$$



1

2

5

Time of computation: 3.010770

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Algebra n. 7

$$\begin{pmatrix} 0 & 0 & \mathbb{E}_1 & 0 \\ 0 & 0 & -\mathbb{E}_2 & 0 \\ -\mathbb{E}_1 & \mathbb{E}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

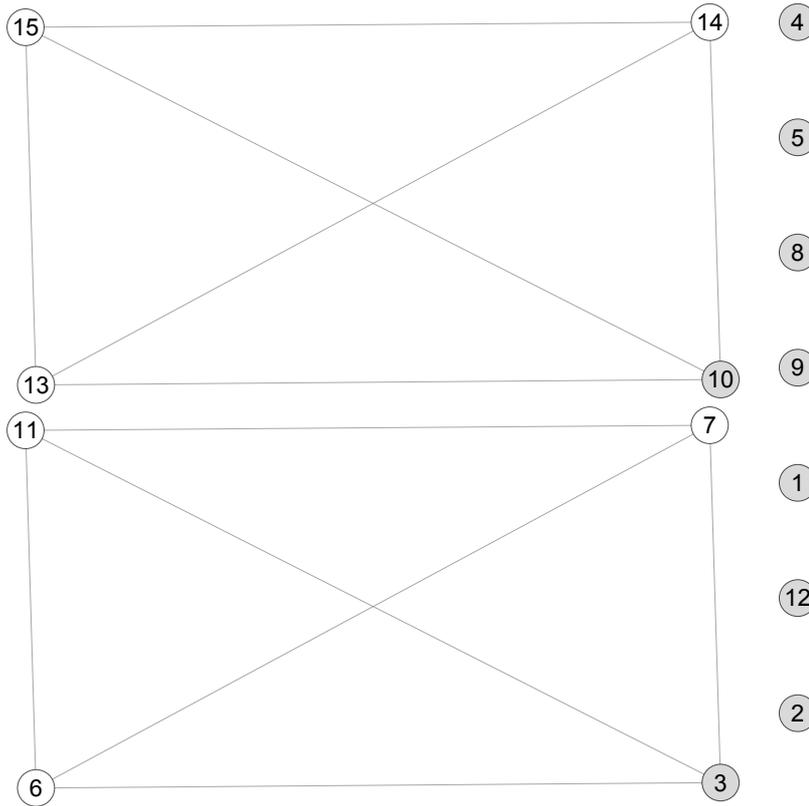
Done.

There are 9 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathbb{E}_1\}, \{\mathbb{E}_2\}, \{\mathbb{E}_3\}, \{\mathbb{E}_4\}, \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_2\}, \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_4\}, \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}, \{\mathbb{E}_3 + \alpha_1 \mathbb{E}_4\}, \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}\}$

$\{1 \rightarrow \{\mathbb{E}_1\}, 2 \rightarrow \{\mathbb{E}_2\}, 3 \rightarrow \{\mathbb{E}_3\}, 4 \rightarrow \{\mathbb{E}_4\}, 5 \rightarrow \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_2\},$

$8 \rightarrow \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_4\}, 9 \rightarrow \{\mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}, 10 \rightarrow \{\mathbb{E}_3 + \alpha_1 \mathbb{E}_4\}, 12 \rightarrow \{\mathbb{E}_1 + \alpha_1 \mathbb{E}_2 + \alpha_1 \mathbb{E}_4\}\}$



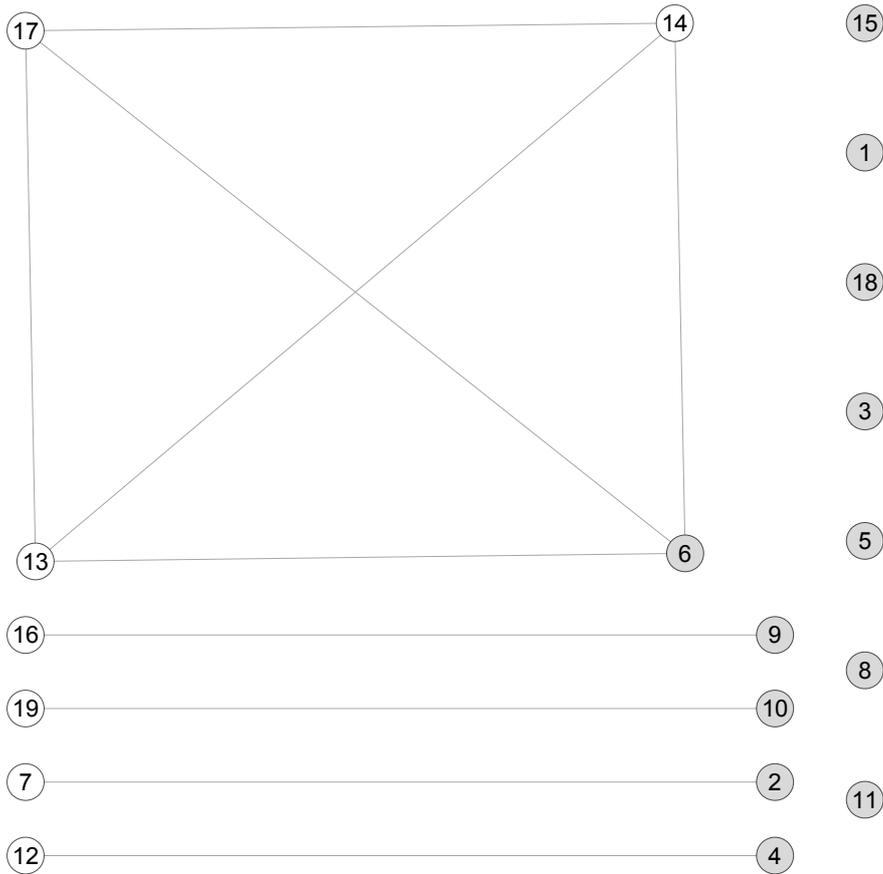
There are 19 2-D families of subalgebras to be analyzed.

Done.

There are 12 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$

$1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\},$
 $6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$
 $11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_4\}, 15 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 18 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$



Time of computation: 2.438456

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Algebra n. 8

$$\begin{pmatrix} 0 & 0 & \mathfrak{E}_1 & 0 \\ 0 & 0 & a \mathfrak{E}_2 & 0 \\ -\mathfrak{E}_1 & -a \mathfrak{E}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

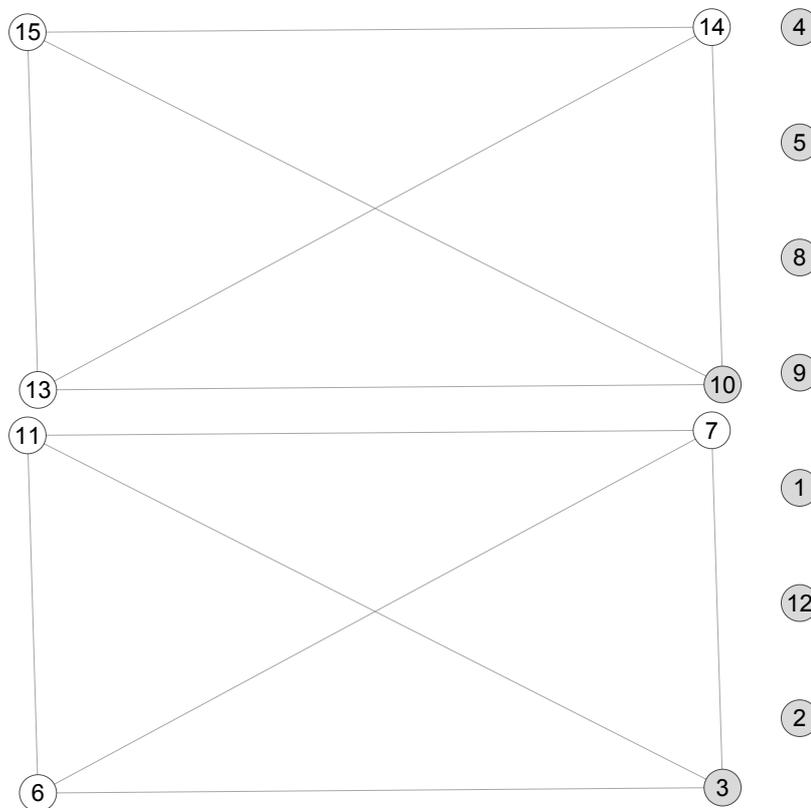
Done.

There are 9 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\},$

$8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$



There are 19 2-D families of subalgebras to be analyzed.

Done.

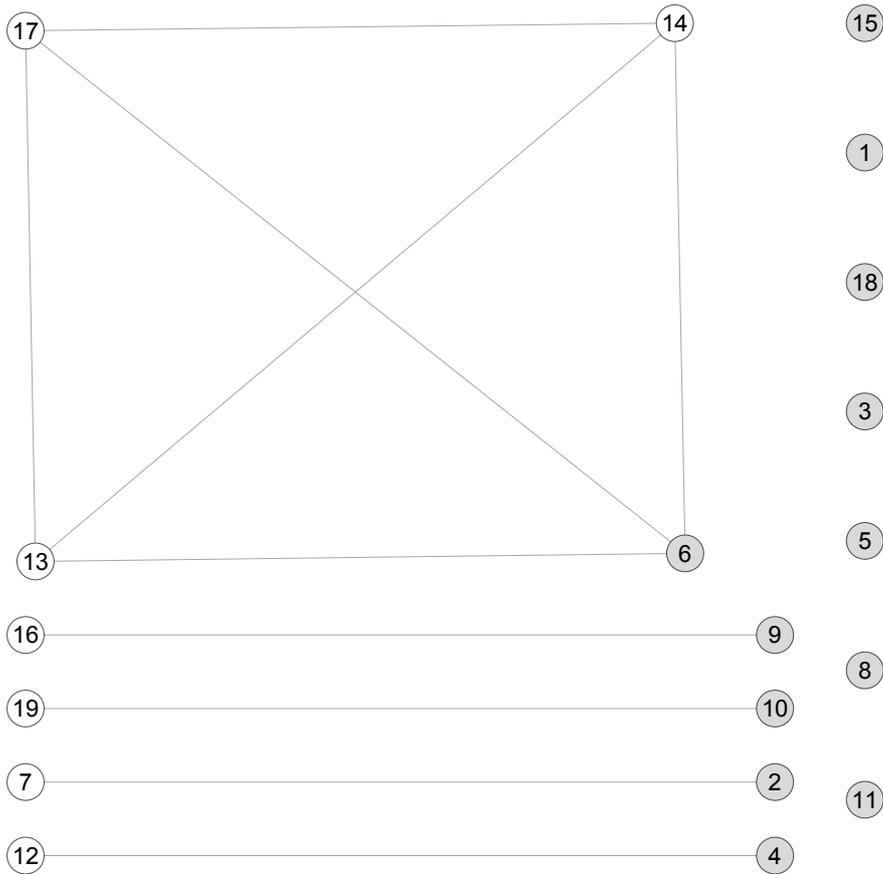
There are 12 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\},$

$6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\},$

$11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_4\}, 15 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 18 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$



Time of computation: 2.544504

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Algebra n. 9

$$\begin{pmatrix} 0 & 0 & -\mathfrak{E}_2 & 0 \\ 0 & 0 & \mathfrak{E}_1 & 0 \\ \mathfrak{E}_2 & -\mathfrak{E}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

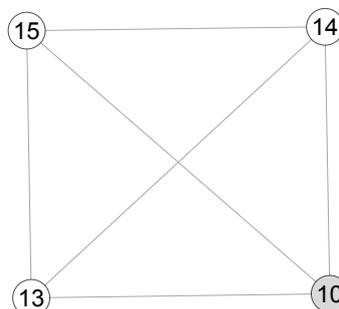
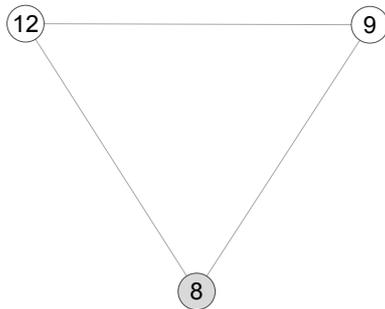
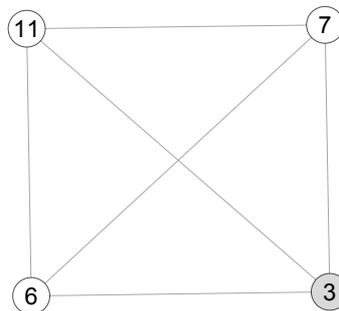
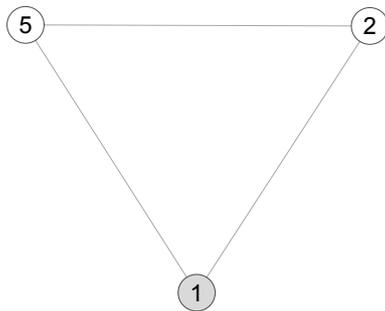
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



4

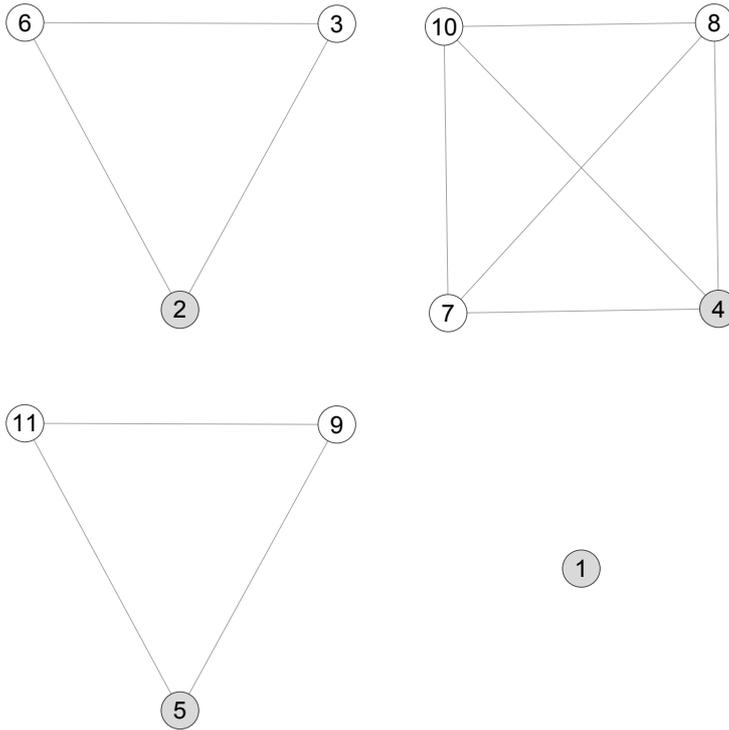
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$$

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②

Time of computation: 9.738252

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Algebra n. 10

$$\begin{pmatrix} 0 & 0 & a \mathfrak{E}_1 - \mathfrak{E}_2 & 0 \\ 0 & 0 & \mathfrak{E}_1 + a \mathfrak{E}_2 & 0 \\ -a \mathfrak{E}_1 + \mathfrak{E}_2 & -\mathfrak{E}_1 - a \mathfrak{E}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

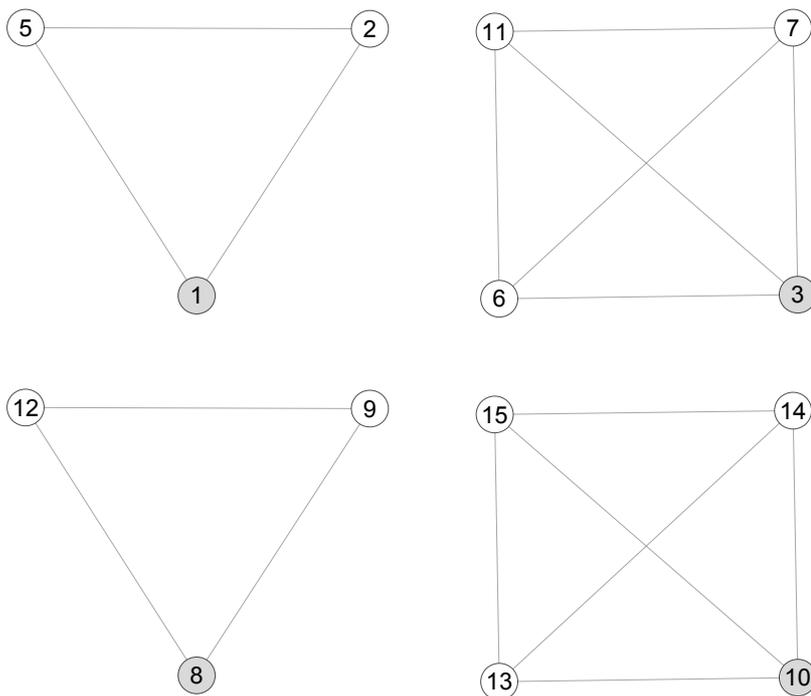
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}, \{\mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$



4

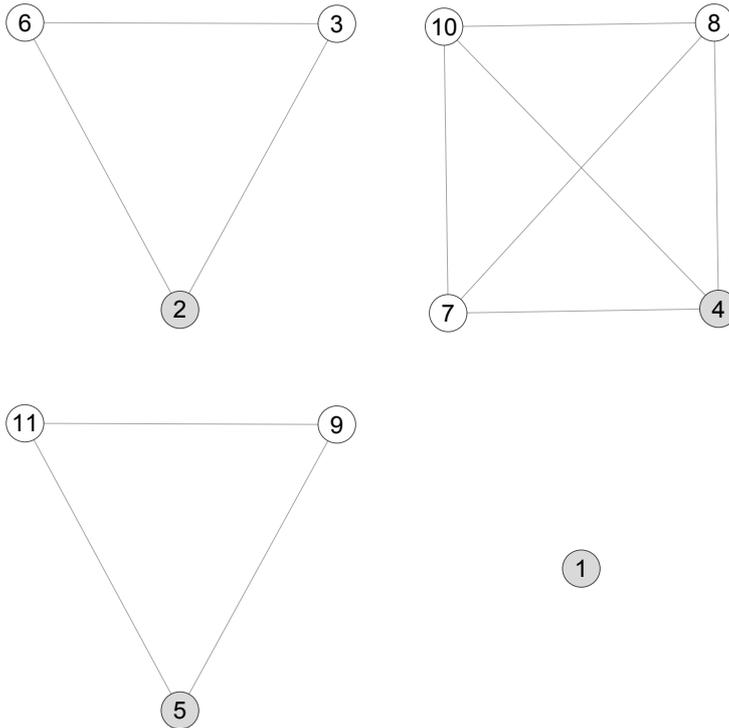
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_4\}\}$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

①

③

②

Time of computation: 57.876057

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Algebra n. 11

$$\begin{pmatrix} 0 & \mathfrak{E}_1 & -2 \mathfrak{E}_2 & 0 \\ -\mathfrak{E}_1 & 0 & \mathfrak{E}_3 & 0 \\ 2 \mathfrak{E}_2 & -\mathfrak{E}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

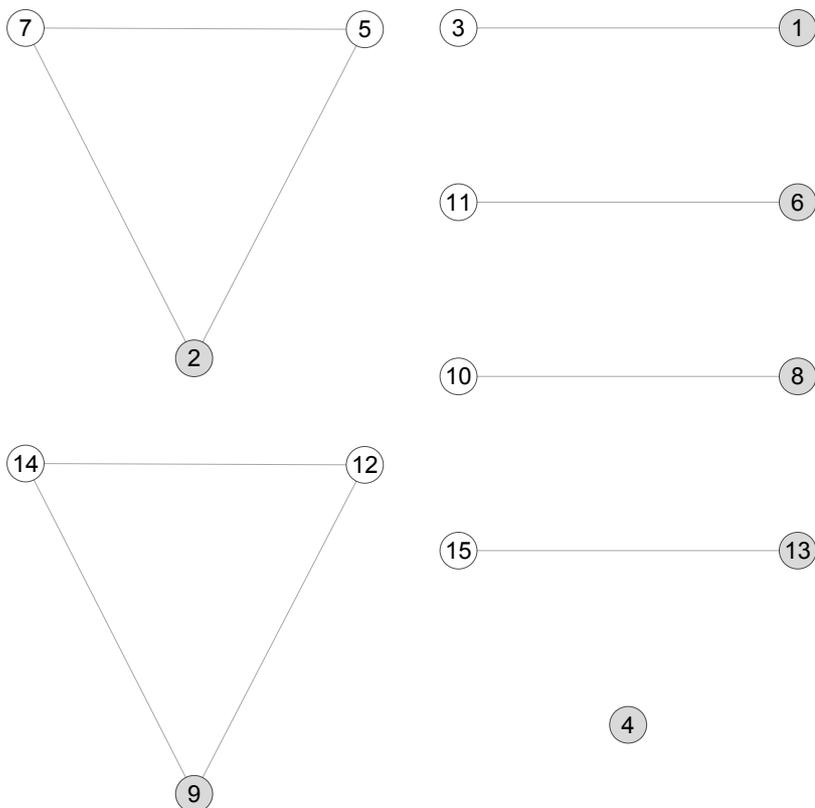
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 7 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 4 \rightarrow \{\mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\},$
 $8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, 13 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$



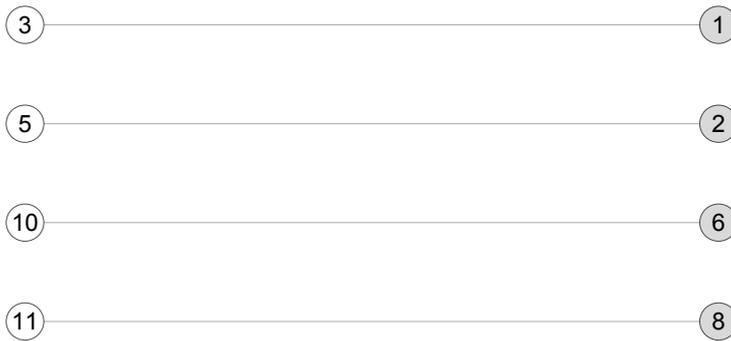
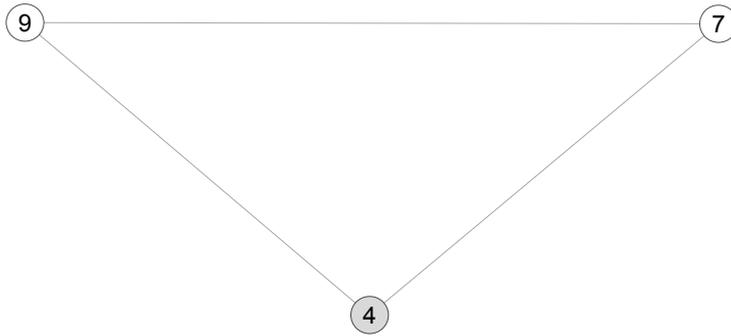
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_4\}\}$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



1

Time of computation: 5.277123

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Algebra n. 12

$$\begin{pmatrix} 0 & \mathfrak{E}_3 & -\mathfrak{E}_2 & 0 \\ -\mathfrak{E}_3 & 0 & \mathfrak{E}_1 & 0 \\ \mathfrak{E}_2 & -\mathfrak{E}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

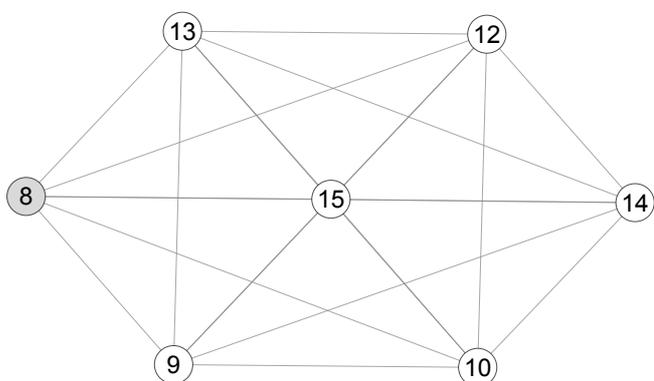
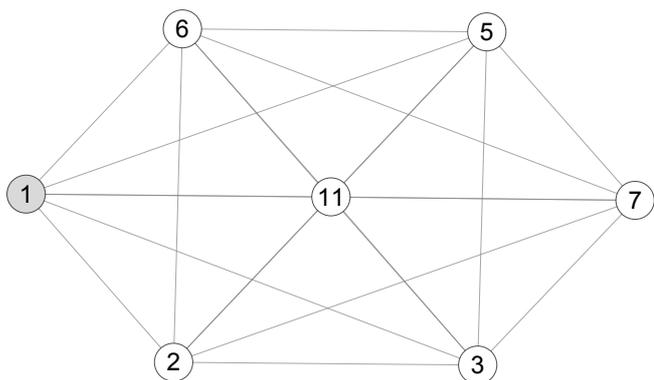
Done.

There are 3 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 4 \rightarrow \{\mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}\}$

4



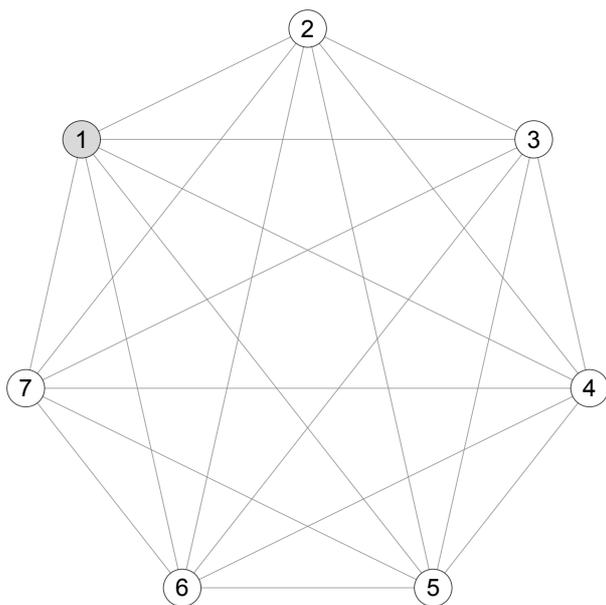
There are 7 2-D families of subalgebras to be analyzed.

Done.

There are 1 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}\}$



There are 1 3-D families of subalgebras to be analyzed.

Done.

There are 1 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$$

Time of computation: 59.427010

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Algebra n. 13

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & -\mathfrak{E}_2 & 0 \end{pmatrix}$$

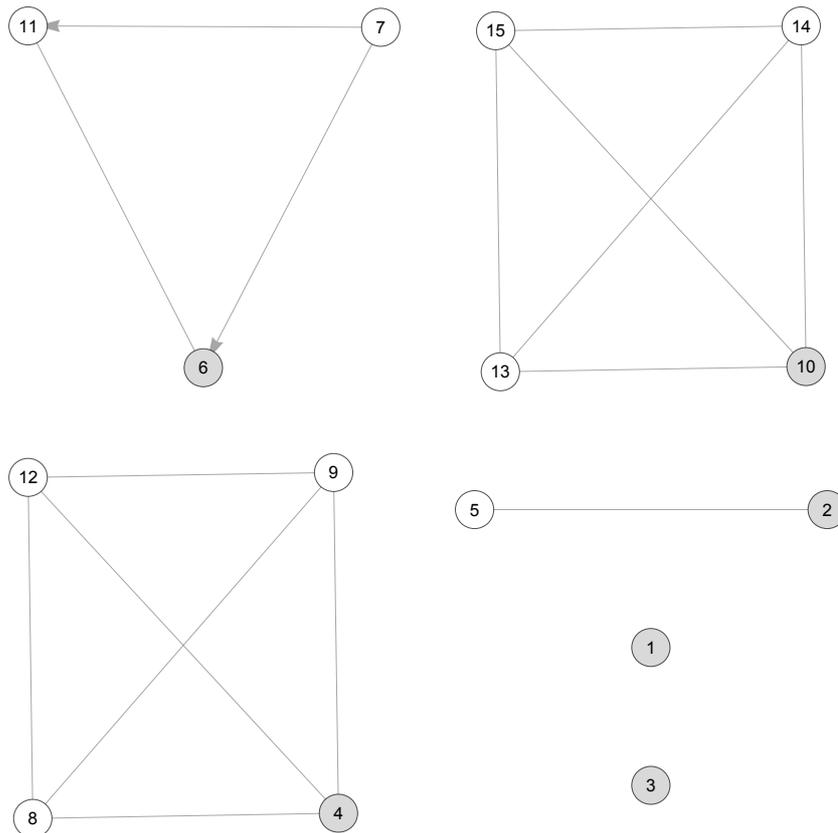
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



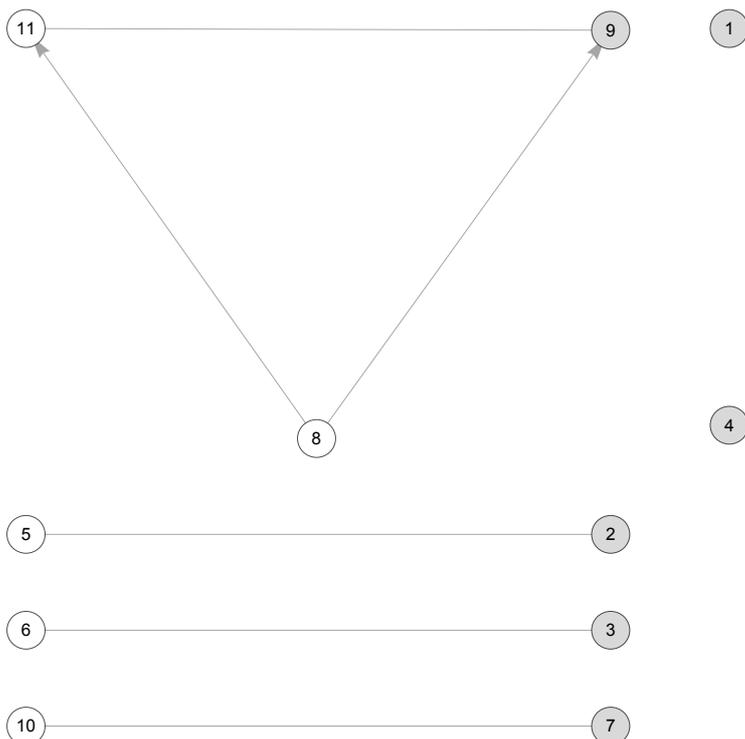
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$

1

3

2

Time of computation: 5.455947

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Algebra n. 14

$$\begin{pmatrix} 0 & 0 & 0 & a \mathfrak{E}_1 \\ 0 & 0 & 0 & \mathfrak{E}_2 \\ 0 & 0 & 0 & \mathfrak{E}_2 + \mathfrak{E}_3 \\ -a \mathfrak{E}_1 & -\mathfrak{E}_2 & -\mathfrak{E}_2 - \mathfrak{E}_3 & 0 \end{pmatrix}$$

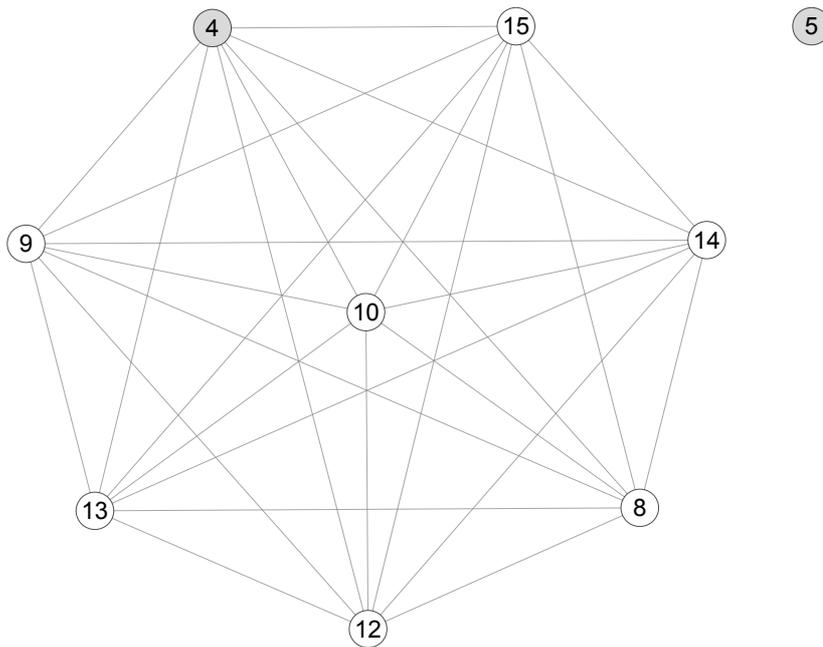
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}\}$$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, 6 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}\}$



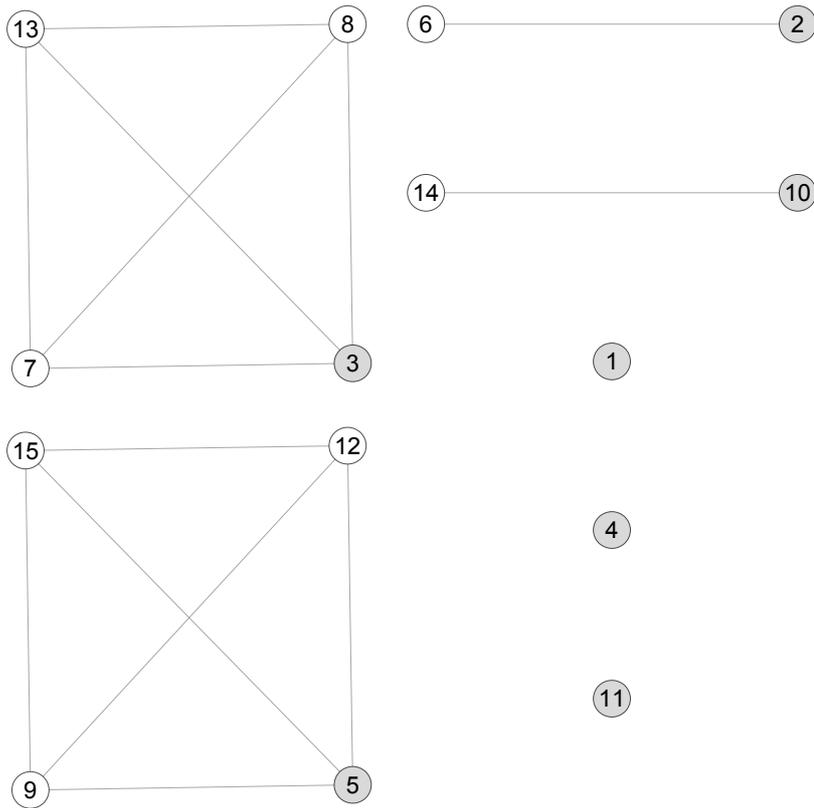
There are 15 2-D families of subalgebras to be analyzed.

Done.

There are 7 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\},$
 $5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$



Time of computation: 37.129170

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Algebra n. 15

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & \mathfrak{E}_2 \\ 0 & 0 & 0 & \mathfrak{E}_2 + \mathfrak{E}_3 \\ -\mathfrak{E}_1 & -\mathfrak{E}_2 & -\mathfrak{E}_2 - \mathfrak{E}_3 & 0 \end{pmatrix}$$

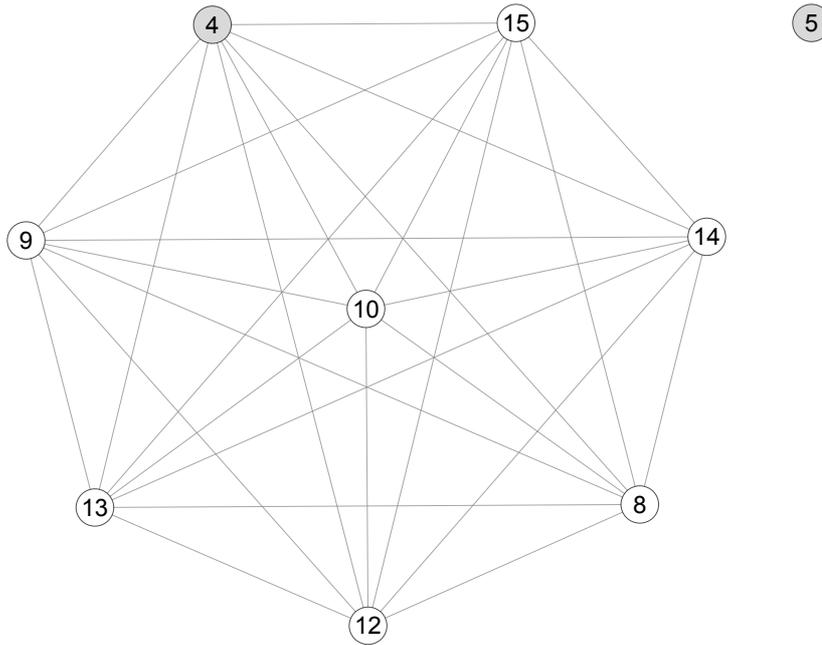
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2\}, 6 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}\}$$



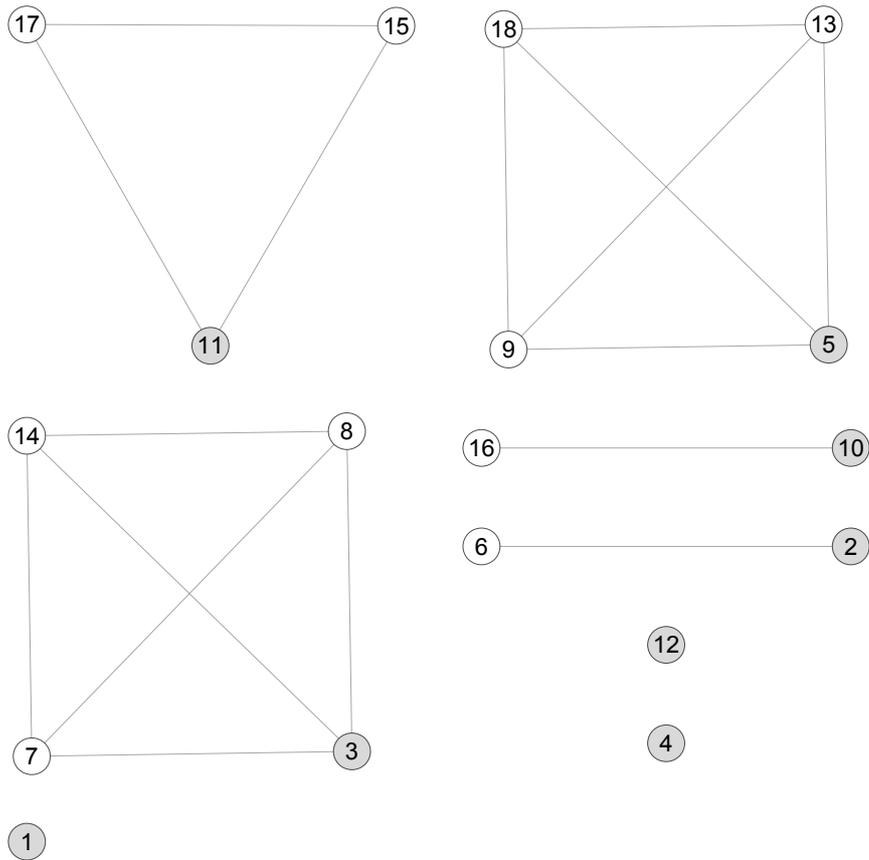
There are 18 2-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



Time of computation: 82.038207

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Algebra n. 16

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{E}_2 \\ -\mathfrak{E}_1 & 0 & -\mathfrak{E}_2 & 0 \end{pmatrix}$$

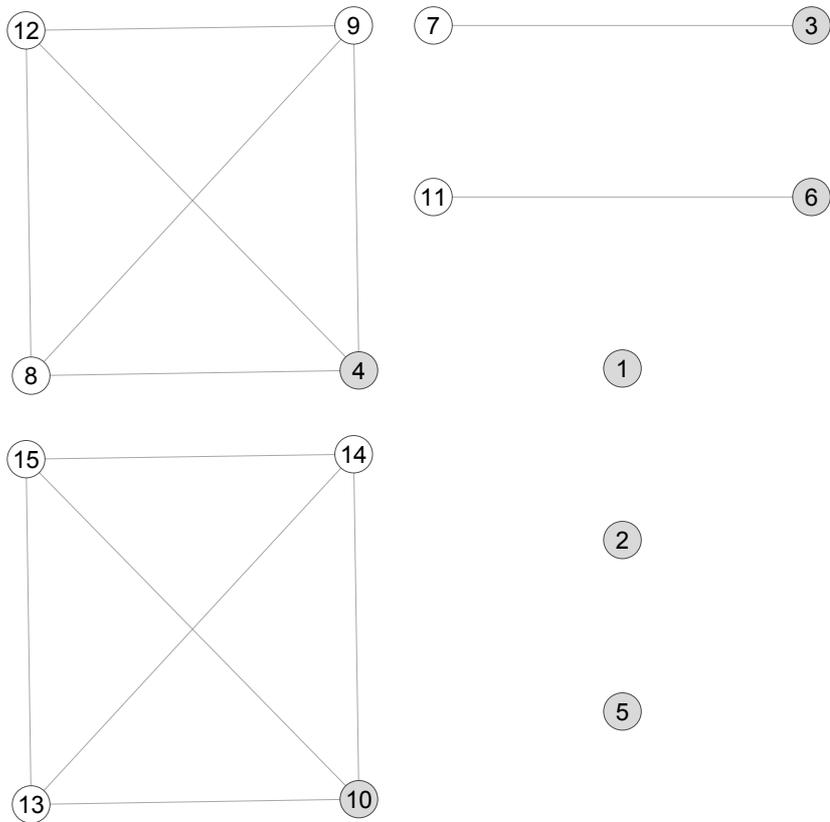
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 7 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, 6 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$$



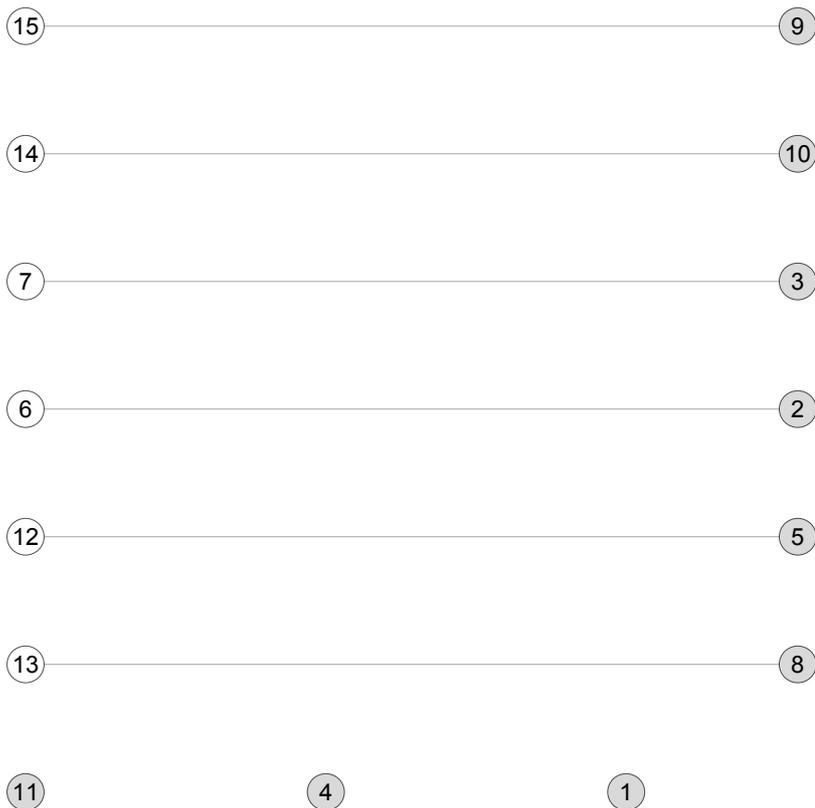
There are 15 2-D families of subalgebras to be analyzed.

Done.

There are 9 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}\}$$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_2, e_3, e_4\}, \{e_1, e_2, e_3 + a_1 e_4\}\}$

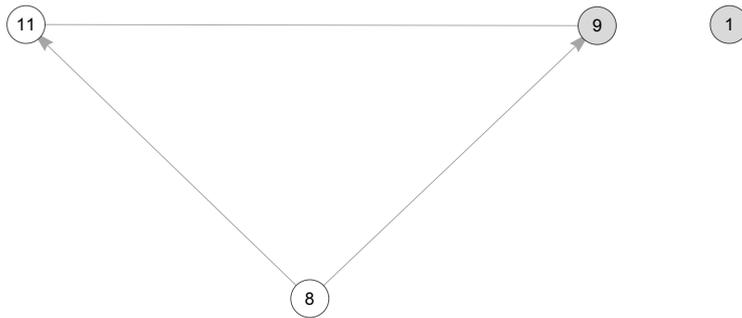
$\{1 \rightarrow \{e_1, e_2, e_3\}, 2 \rightarrow \{e_1, e_2, e_4\}, 3 \rightarrow \{e_2, e_3, e_4\}, 4 \rightarrow \{e_1, e_2, e_3 + a_1 e_4\}\}$



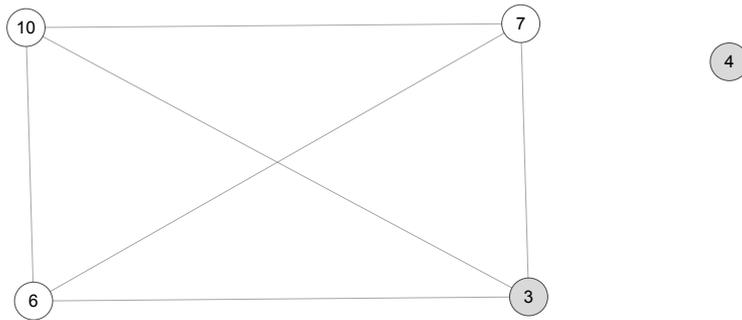
Time of computation: 9.039540

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Algebra n. 17



1



4



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



1

Time of computation: 46.033450

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Algebra n. 18

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & a \mathfrak{E}_2 \\ 0 & 0 & 0 & b \mathfrak{E}_3 \\ -\mathfrak{E}_1 & -a \mathfrak{E}_2 & -b \mathfrak{E}_3 & 0 \end{pmatrix}$$

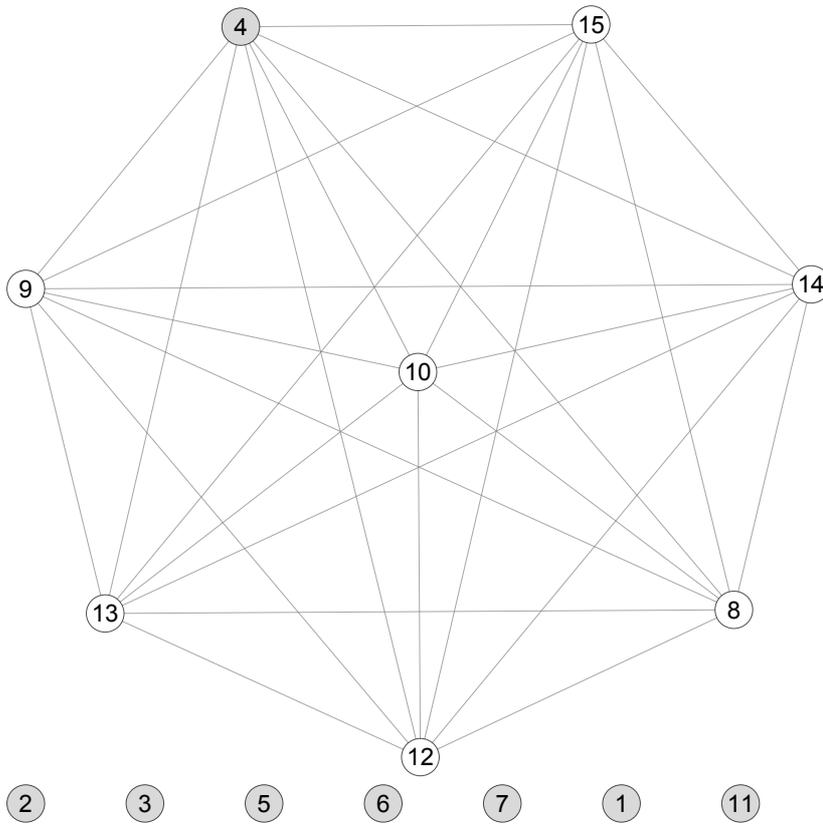
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\},$
 $6 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\}$



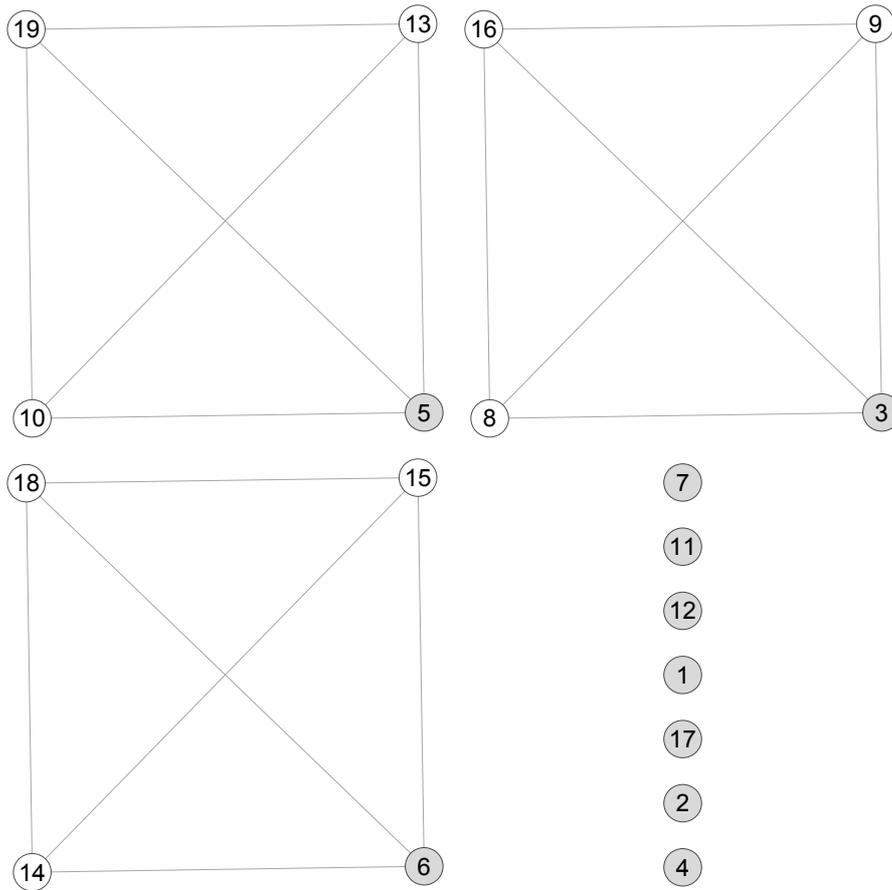
There are 19 2-D families of subalgebras to be analyzed.

Done.

There are 10 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\},$
 $\{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\},$
 $7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, 12 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 17 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2 + \mathfrak{a}_1 \mathfrak{E}_3\}\}$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$



Time of computation: 5.847802

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Algebra n. 19

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & a \mathfrak{E}_2 \\ 0 & 0 & 0 & a \mathfrak{E}_3 \\ -\mathfrak{E}_1 & -a \mathfrak{E}_2 & -a \mathfrak{E}_3 & 0 \end{pmatrix}$$

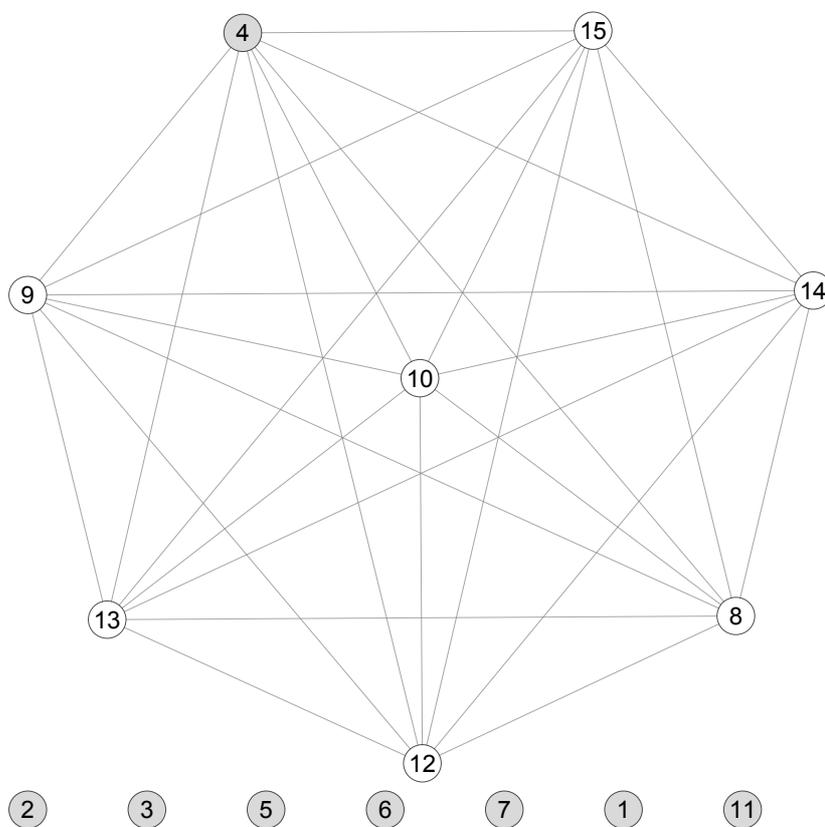
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \\ 6 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$$



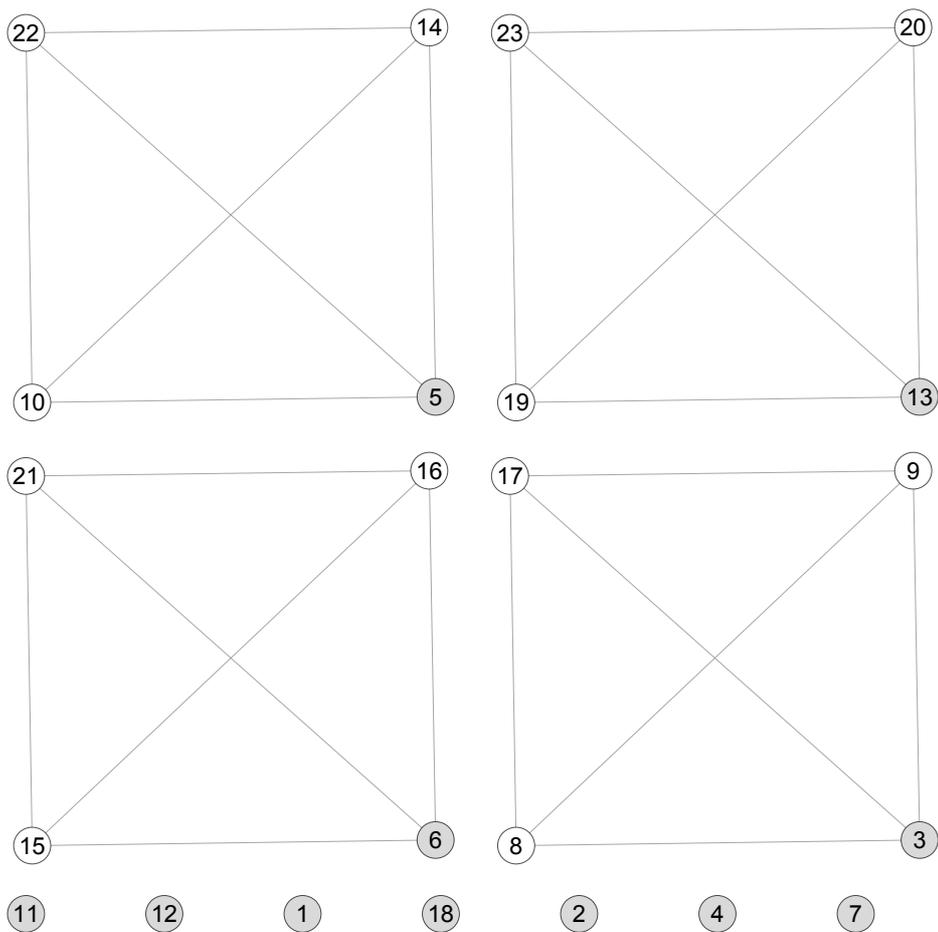
There are 23 2-D families of subalgebras to be analyzed.

Done.

There are 11 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, \\ \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, \\ 5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \\ 12 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 13 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 18 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$$



There are 9 3-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}$



1

Time of computation: 8.774573

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Algebra n. 20

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & a \mathfrak{E}_2 \\ 0 & 0 & 0 & \mathfrak{E}_3 \\ -\mathfrak{E}_1 & -a \mathfrak{E}_2 & -\mathfrak{E}_3 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

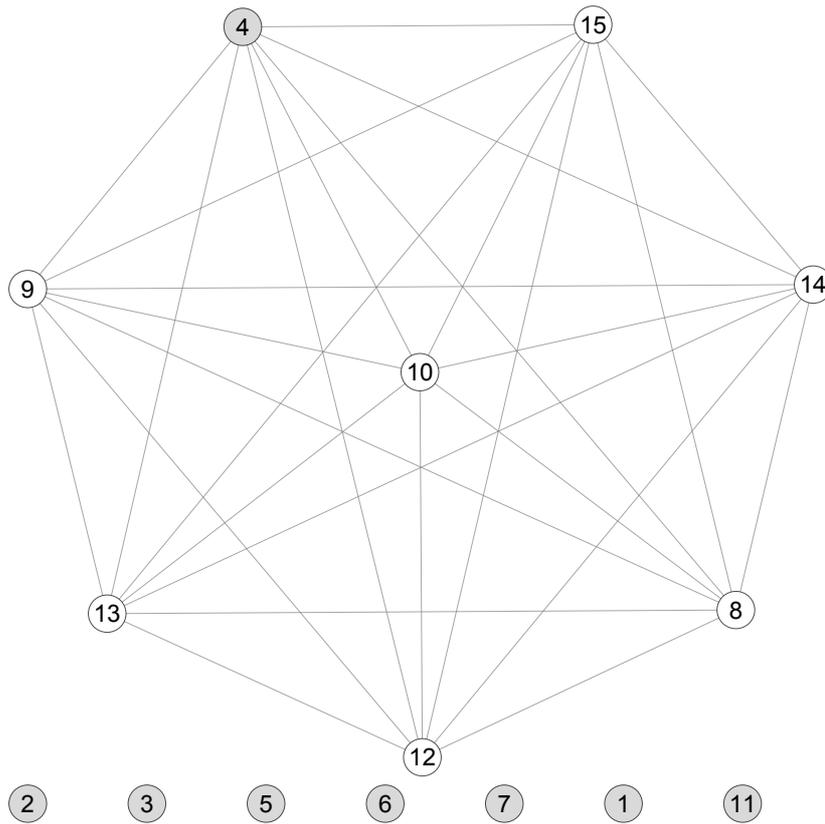
Done.

There are 8 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2\},$

$6 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$



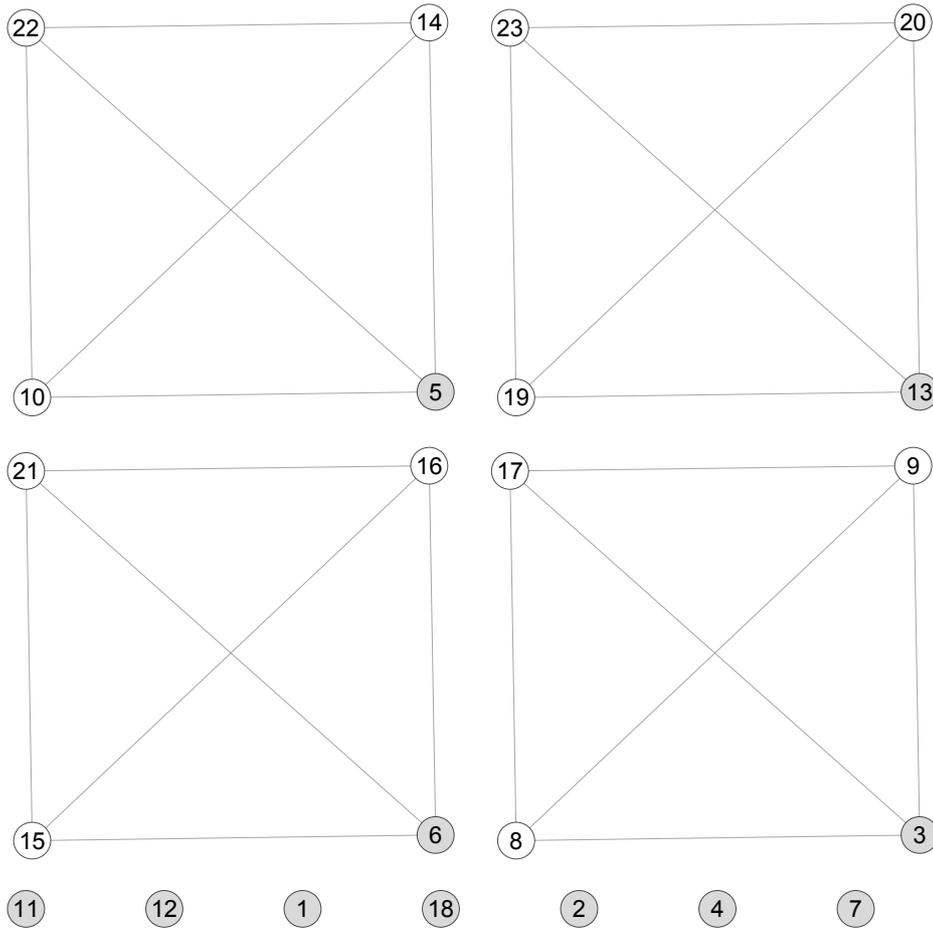
There are 23 2-D families of subalgebras to be analyzed.

Done.

There are 11 optimal families of 2-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\},$
 $\{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}$

$1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\},$
 $5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_2, \mathfrak{E}_3\},$
 $12 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 13 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 18 \rightarrow \{\mathfrak{E}_1 + \mathfrak{a}_1 \mathfrak{E}_3, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}$



There are 9 3-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



1

Time of computation: 7.689911

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Algebra n. 21

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & 0 & \mathfrak{E}_2 \\ 0 & 0 & 0 & \mathfrak{E}_3 \\ -\mathfrak{E}_1 & -\mathfrak{E}_2 & -\mathfrak{E}_3 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

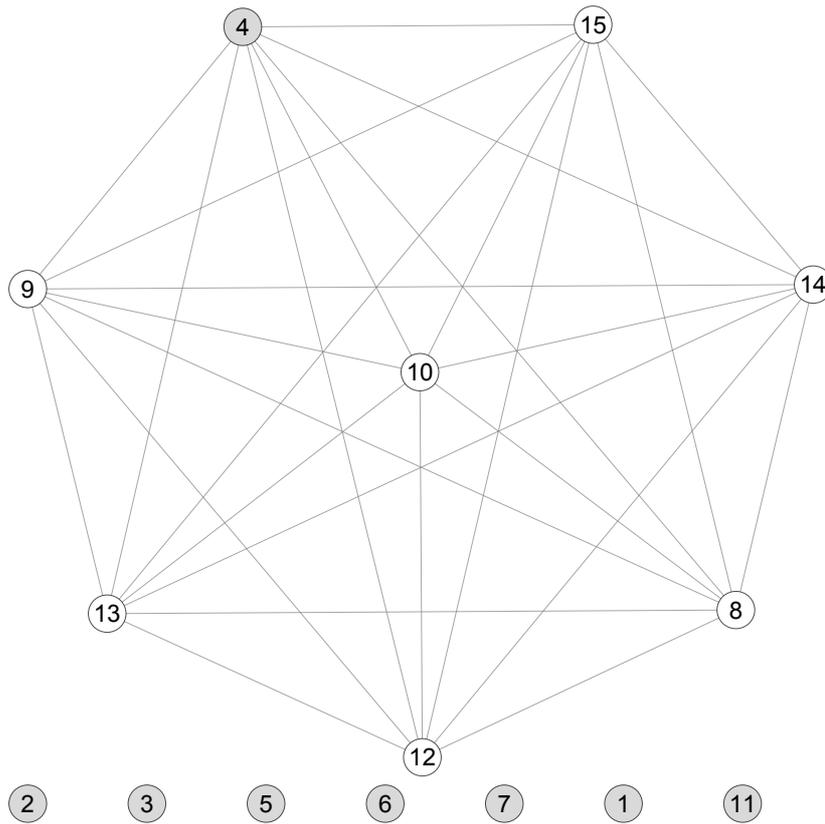
Done.

There are 8 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2 + a_2 \mathfrak{E}_3\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2\},$

$6 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3\}, 7 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2 + a_2 \mathfrak{E}_3\}\}$

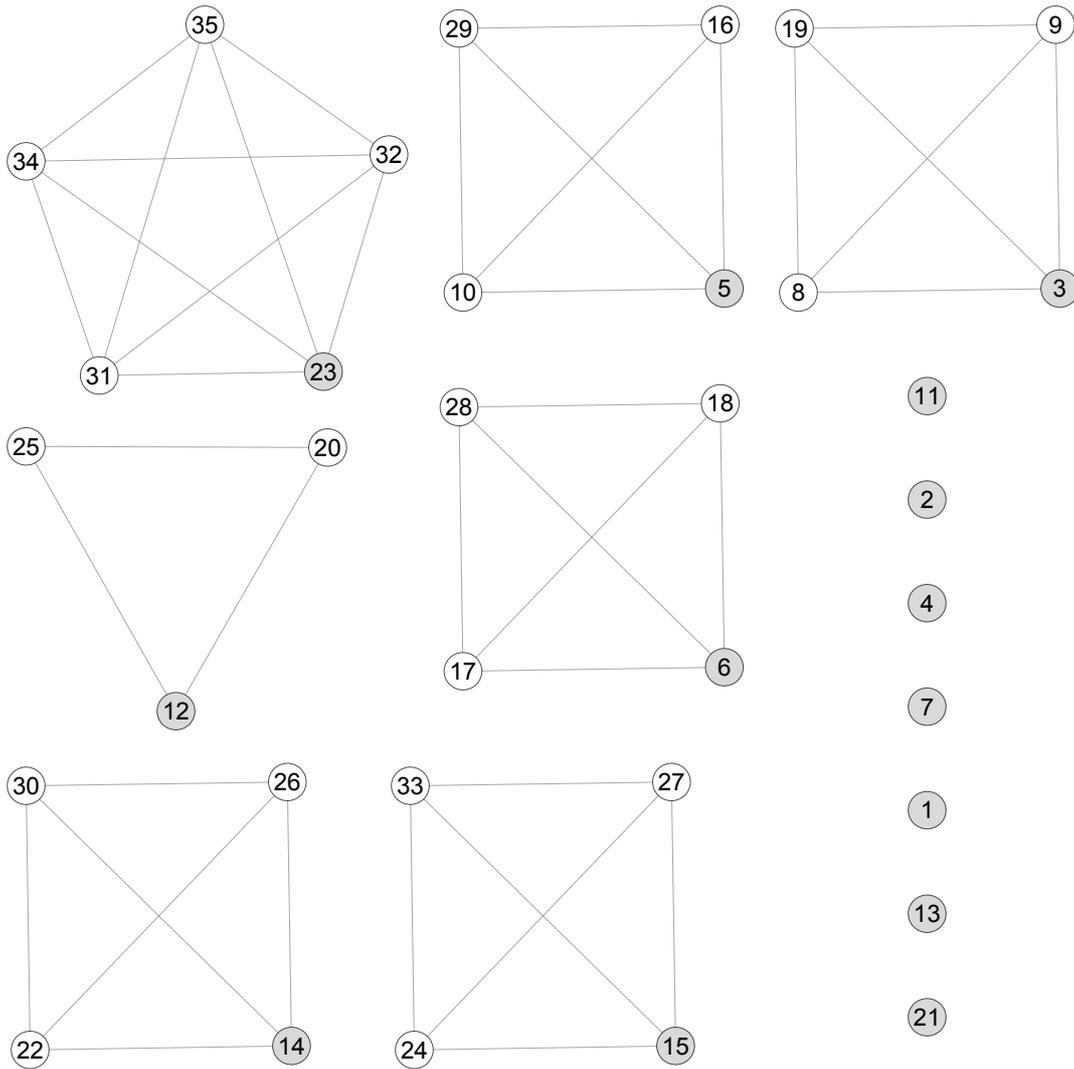


There are 35 2-D families of subalgebras to be analyzed.

Done.

There are 14 optimal families of 2-dimensional Lie subalgebras.

- $\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\},$
- $\{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2\},$
- $\{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_2 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2 + a_2 \mathfrak{E}_3, \mathfrak{E}_4\}$
- $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\},$
- $5 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, 11 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3\},$
- $12 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_4\}, 13 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2\}, 14 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\},$
- $15 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, 21 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_2 \mathfrak{E}_3\}, 23 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2 + a_2 \mathfrak{E}_3, \mathfrak{E}_4\}$



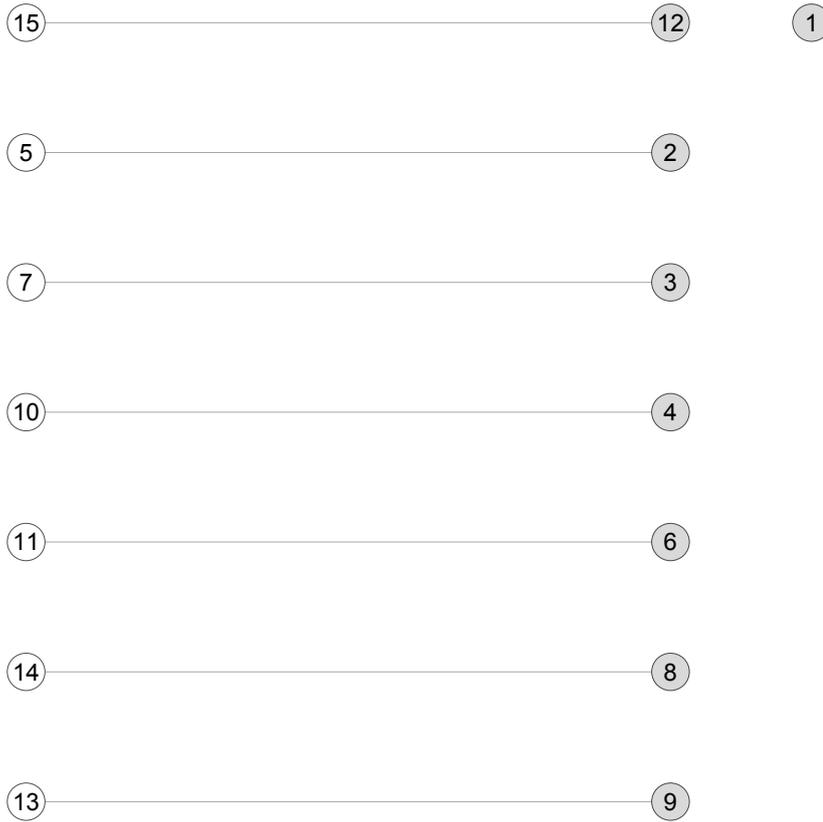
There are 15 3-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \\ \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_2 \mathfrak{E}_3, \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}, \\ 8 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}, 9 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2, \mathfrak{E}_4\}, 12 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_3, \mathfrak{E}_2 + a_2 \mathfrak{E}_3, \mathfrak{E}_4\}\}$$



Time of computation: 23.635729

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Algebra n. 22

$$\begin{pmatrix} 0 & 0 & 0 & a \mathfrak{E}_1 \\ 0 & 0 & 0 & b \mathfrak{E}_2 - \mathfrak{E}_3 \\ 0 & 0 & 0 & \mathfrak{E}_2 + b \mathfrak{E}_3 \\ -a \mathfrak{E}_1 & -b \mathfrak{E}_2 + \mathfrak{E}_3 & -\mathfrak{E}_2 - b \mathfrak{E}_3 & 0 \end{pmatrix}$$

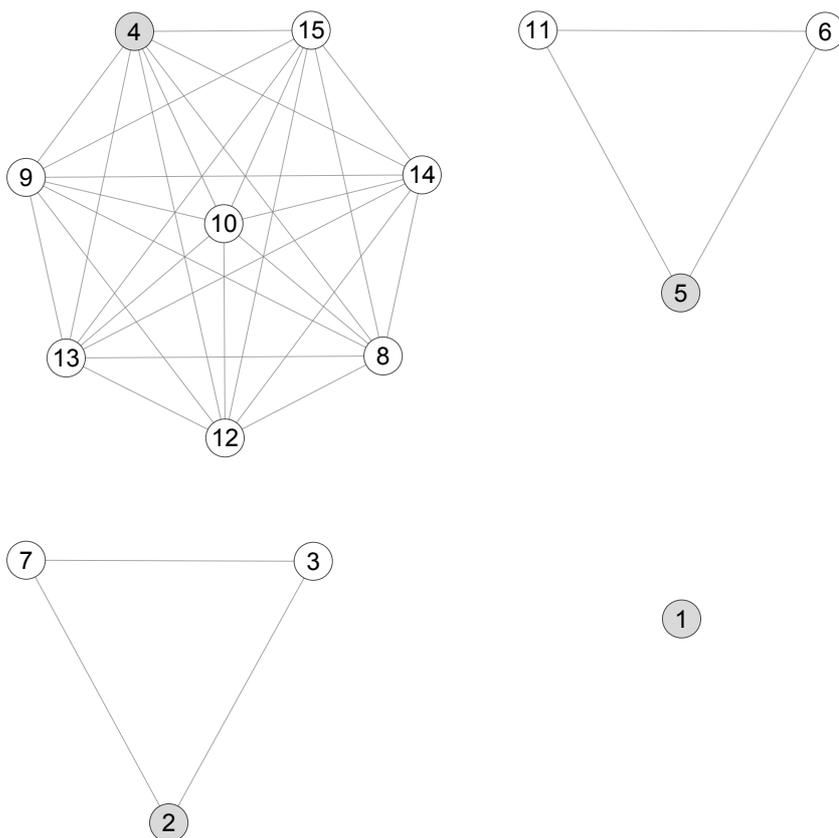
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 4 \rightarrow \{\mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2\}\}$



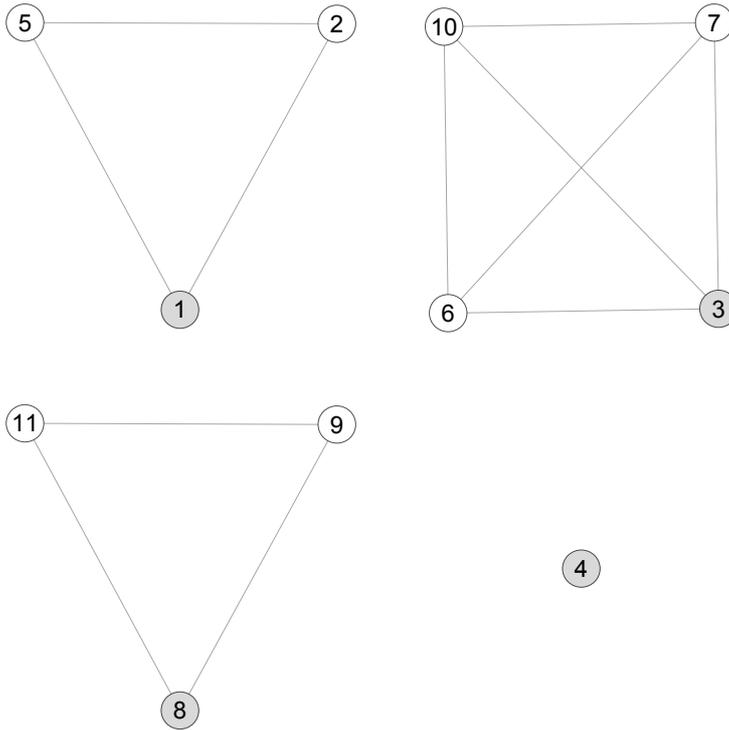
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 2-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3\}, 8 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_2, \mathfrak{E}_3\}\}$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 3-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4\}\}$$



①

Time of computation: 152.251413

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Algebra n. 23

$$\begin{pmatrix} 0 & 0 & 0 & 2\mathfrak{E}_1 \\ 0 & 0 & \mathfrak{E}_1 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & 0 & \mathfrak{E}_2 + \mathfrak{E}_3 \\ -2\mathfrak{E}_1 & -\mathfrak{E}_2 & -\mathfrak{E}_2 - \mathfrak{E}_3 & 0 \end{pmatrix}$$

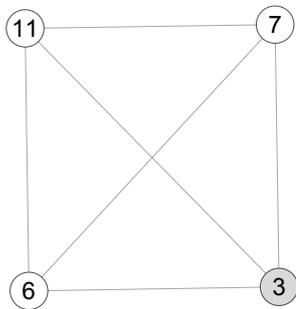
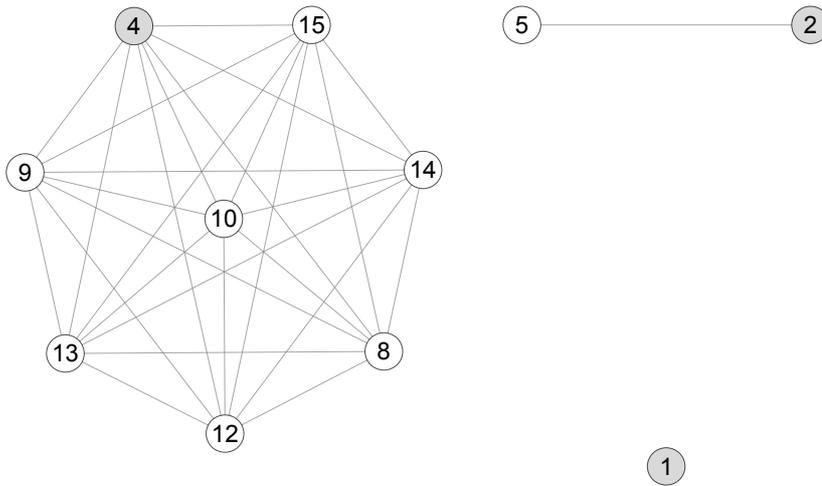
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}\}$$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}\}$



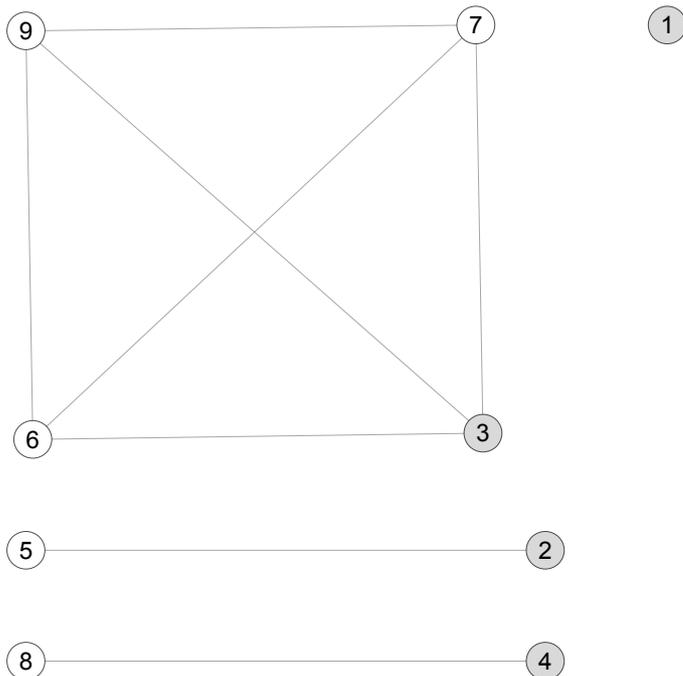
There are 9 2-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}\}$



There are 3 3-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}\}$



①

Time of computation: 82.807977

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Algebra n. 24

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mathfrak{E}_1 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & 0 & -\mathfrak{E}_3 \\ 0 & -\mathfrak{E}_2 & \mathfrak{E}_3 & 0 \end{pmatrix}$$

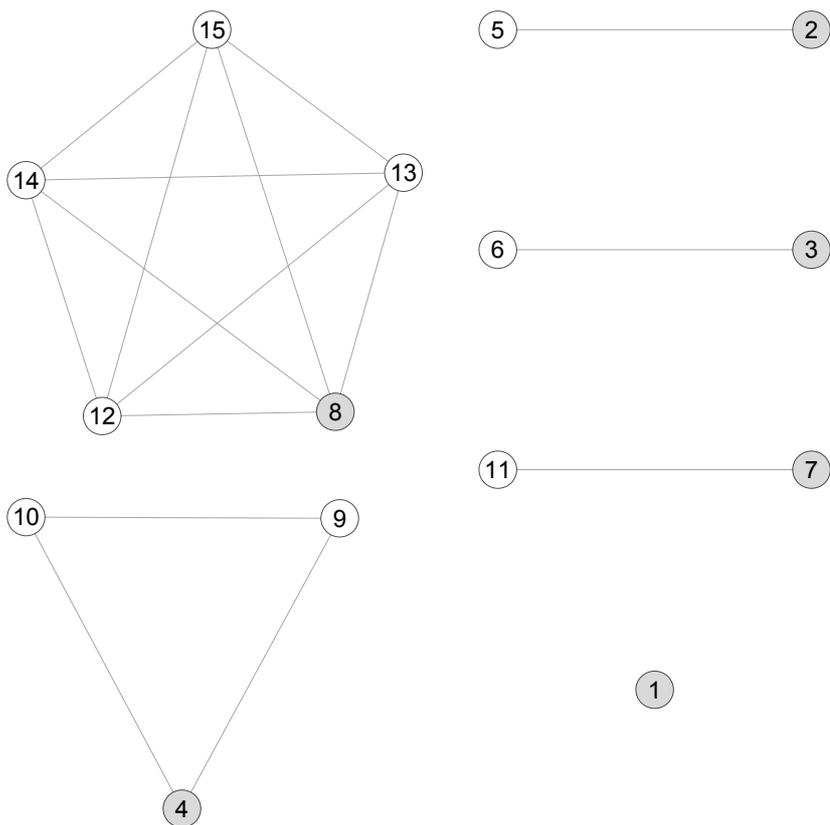
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 8 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4\}\}$

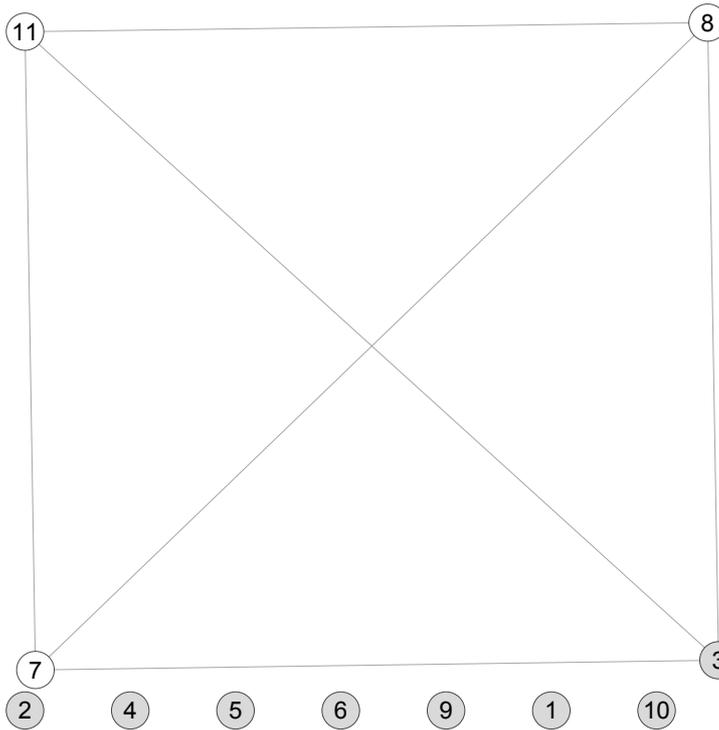


There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 8 optimal families of 2-dimensional Lie subalgebras.

- $\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\},$
- $\{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3\}$
- $1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\},$
- $5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 9 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_2\}, 10 \rightarrow \{\mathfrak{E}_1 + \alpha_1 \mathfrak{E}_4, \mathfrak{E}_3\}$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}$

$1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}$



Time of computation: 3.801006

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Algebra n. 25

$$\begin{pmatrix} 0 & 0 & 0 & (1+b)\mathfrak{E}_1 \\ 0 & 0 & \mathfrak{E}_1 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & 0 & b\mathfrak{E}_3 \\ (-1-b)\mathfrak{E}_1 & -\mathfrak{E}_2 & -b\mathfrak{E}_3 & 0 \end{pmatrix}$$

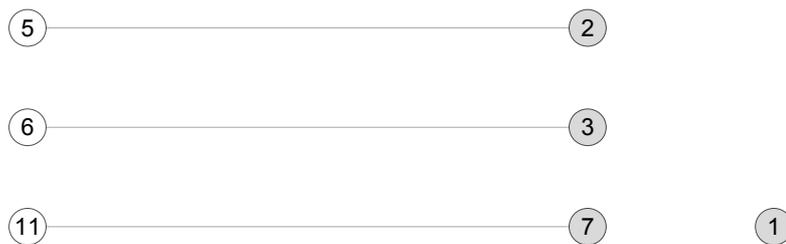
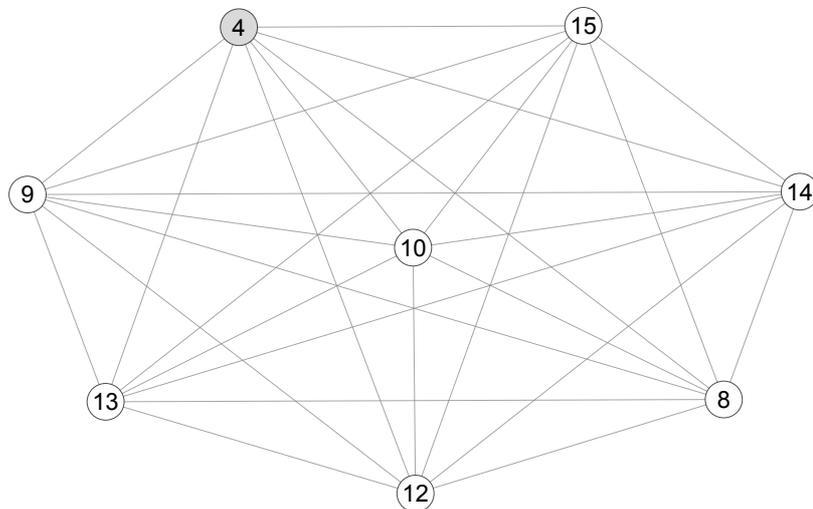
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 1-dimensional Lie subalgebras.

$$\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}\}$$



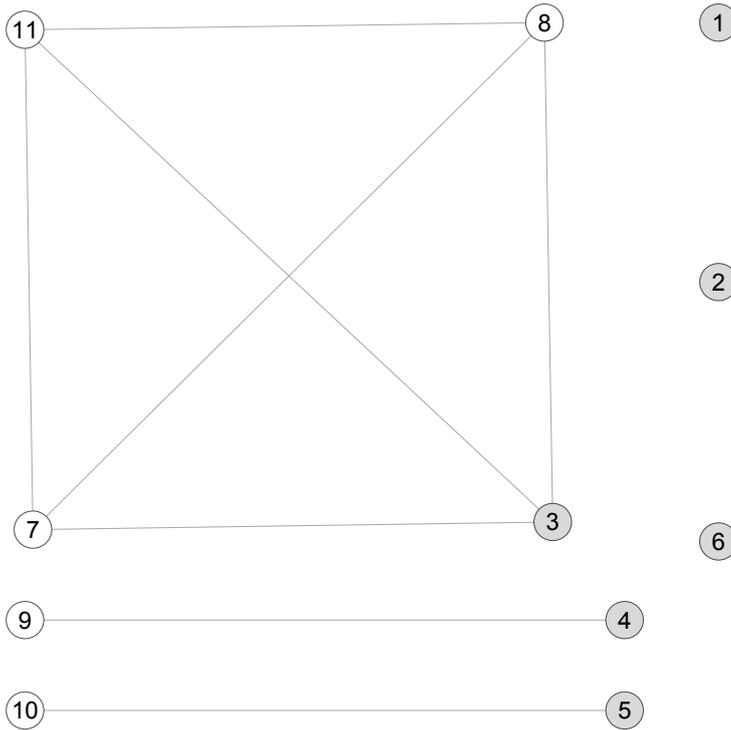
There are 11 2-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 2-dimensional Lie subalgebras.

$$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}\}$$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}$

$1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}$



Time of computation: 8.018379

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Algebra n. 26

$$\begin{pmatrix} 0 & 0 & 0 & 2\mathfrak{E}_1 \\ 0 & 0 & \mathfrak{E}_1 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & 0 & \mathfrak{E}_3 \\ -2\mathfrak{E}_1 & -\mathfrak{E}_2 & -\mathfrak{E}_3 & 0 \end{pmatrix}$$

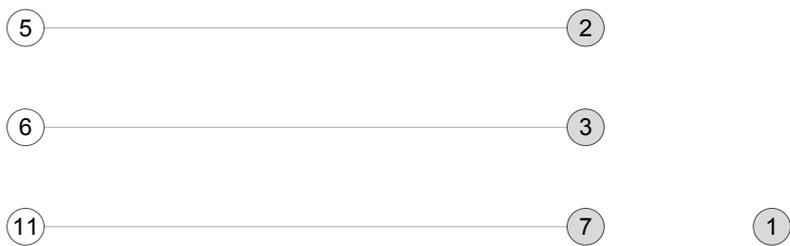
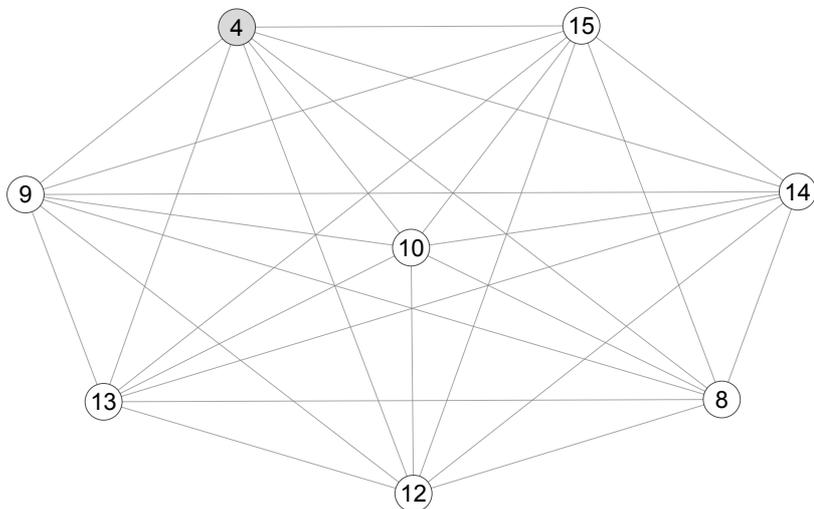
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 5 optimal families of 1-dimensional Lie subalgebras.

$$\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3\}\}$$



There are 13 2-D families of subalgebras to be analyzed.

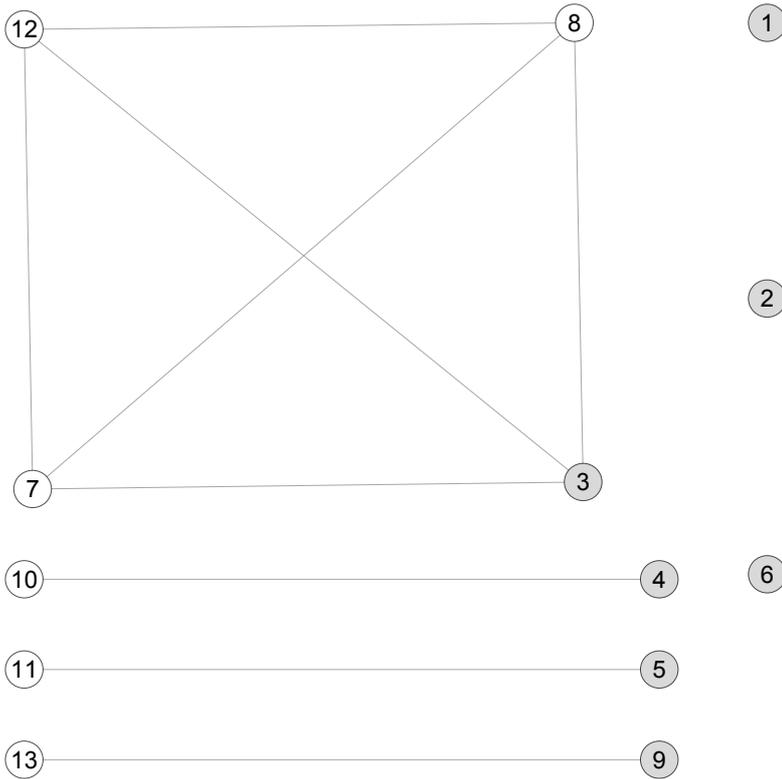
Done.

There are 7 optimal families of 2-dimensional Lie subalgebras.

$$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\},$$

$$4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3\}, 9 \rightarrow \{\mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}\}$$



There are 7 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + a_1 \mathfrak{E}_3, \mathfrak{E}_4\}\}$



Time of computation: 11.168949

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Algebra n. 27

$$\begin{pmatrix} 0 & 0 & 0 & \mathfrak{E}_1 \\ 0 & 0 & \mathfrak{E}_1 & \mathfrak{E}_2 \\ 0 & -\mathfrak{E}_1 & 0 & 0 \\ -\mathfrak{E}_1 & -\mathfrak{E}_2 & 0 & 0 \end{pmatrix}$$

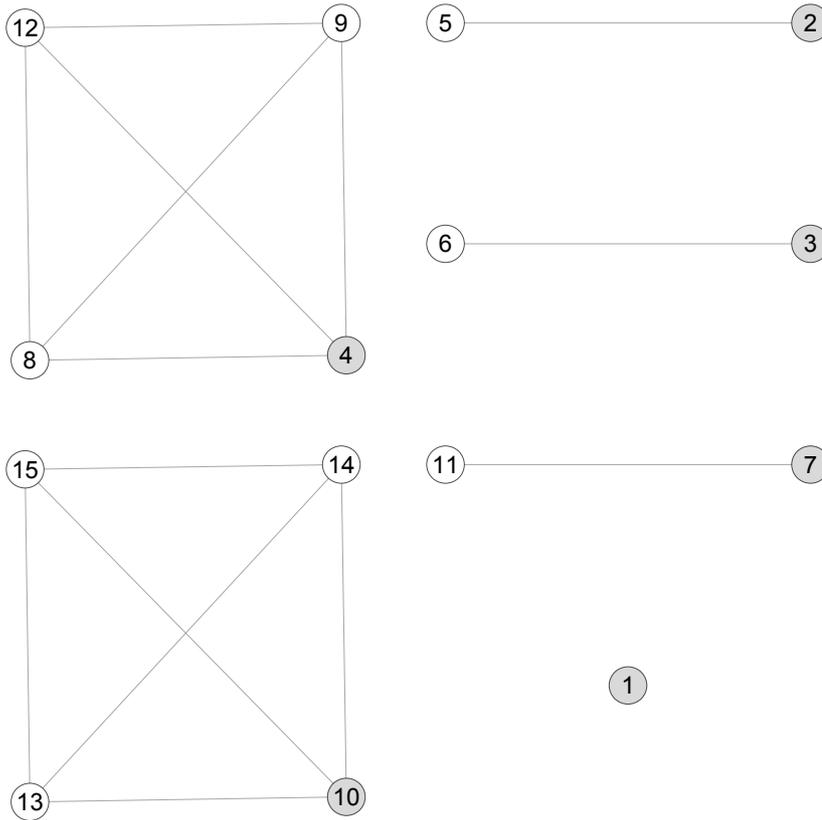
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 6 optimal families of 1-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 7 \rightarrow \{\mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 10 \rightarrow \{\mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$



There are 13 2-D families of subalgebras to be analyzed.

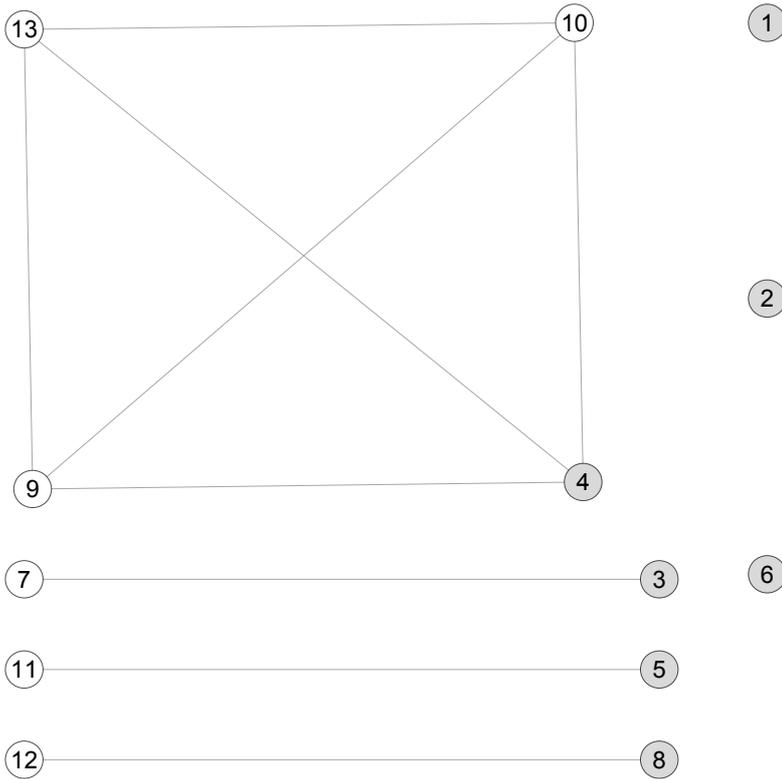
Done.

There are 7 optimal families of 2-dimensional Lie subalgebras.

$$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}, \{\mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$

$$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\},$$

$$4 \rightarrow \{\mathfrak{E}_2, \mathfrak{E}_4\}, 5 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}, 6 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2 + \alpha_1 \mathfrak{E}_3\}, 8 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3 + \alpha_1 \mathfrak{E}_4\}\}$$



There are 5 3-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_4\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3, \mathfrak{E}_4\}, 4 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3 + a_1 \mathfrak{E}_4\}\}$



Time of computation: 7.696302

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Algebra n. 28

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mathfrak{E}_1 & -\mathfrak{E}_3 \\ 0 & -\mathfrak{E}_1 & 0 & \mathfrak{E}_2 \\ 0 & \mathfrak{E}_3 & -\mathfrak{E}_2 & 0 \end{pmatrix}$$

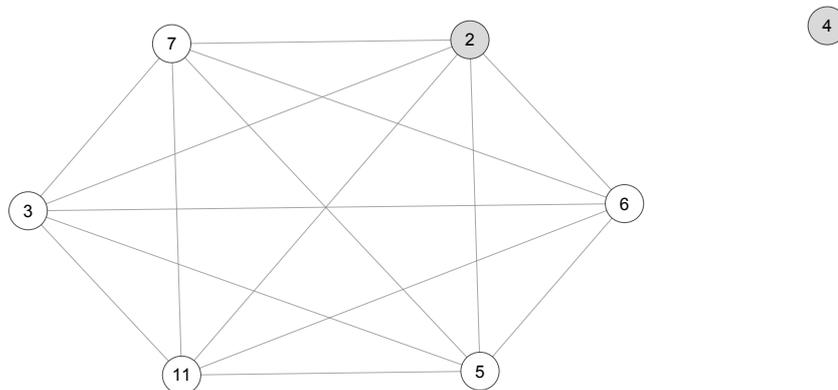
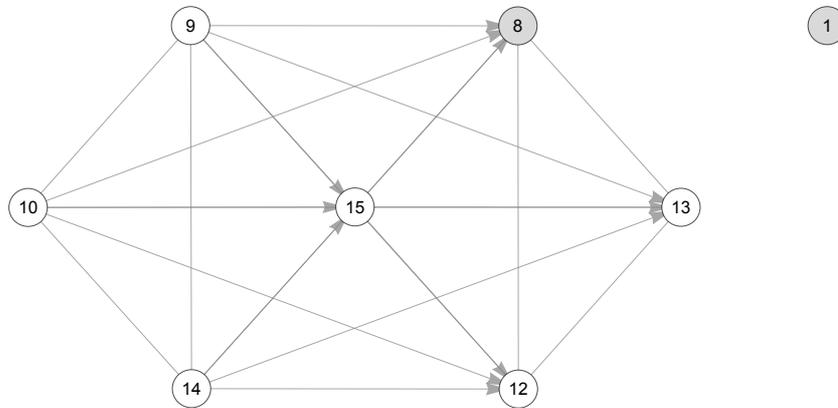
There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 4 \rightarrow \{\mathfrak{E}_4\}, 8 \rightarrow \{\mathfrak{E}_1 + a_1 \mathfrak{E}_4\}\}$



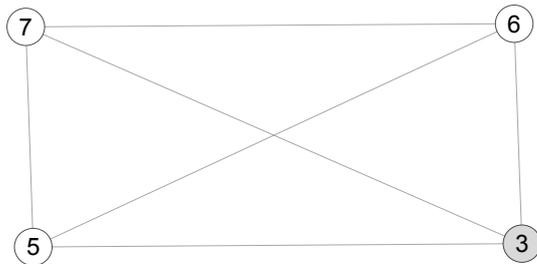
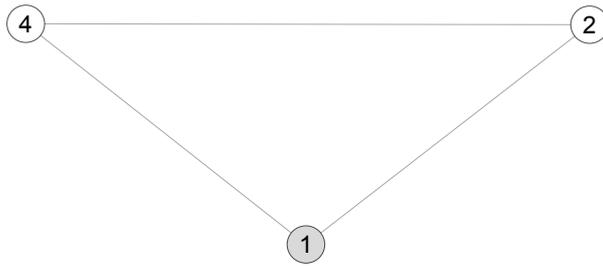
There are 7 2-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}\}$



There are 1 3-D families of subalgebras to be analyzed.

Done.

There are 1 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$

Time of computation: 56.414244

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Algebra n. 29

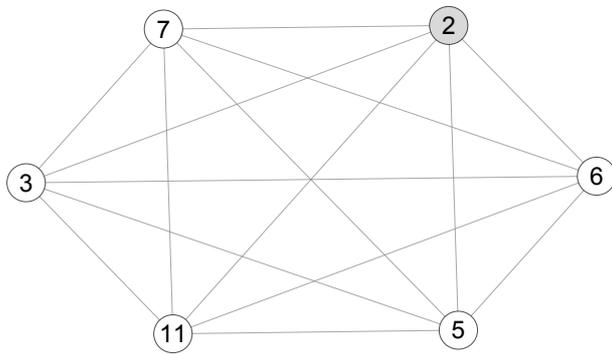
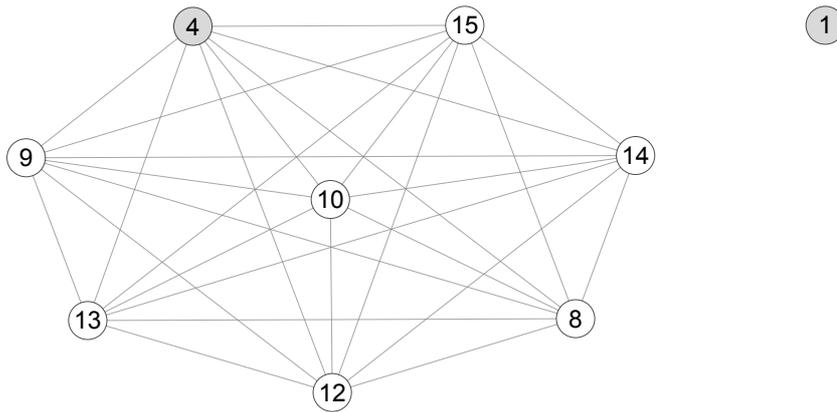
$$\begin{pmatrix} 0 & 0 & 0 & 2a\mathfrak{E}_1 \\ 0 & 0 & \mathfrak{E}_1 & a\mathfrak{E}_2 - \mathfrak{E}_3 \\ 0 & -\mathfrak{E}_1 & 0 & \mathfrak{E}_2 + a\mathfrak{E}_3 \\ -2a\mathfrak{E}_1 & -a\mathfrak{E}_2 + \mathfrak{E}_3 & -\mathfrak{E}_2 - a\mathfrak{E}_3 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_2\}, \{\mathfrak{E}_4\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1\}, 2 \rightarrow \{\mathfrak{E}_2\}, 4 \rightarrow \{\mathfrak{E}_4\}\}$



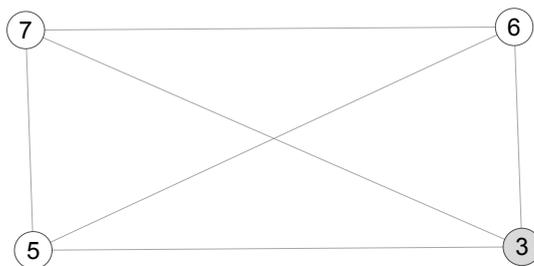
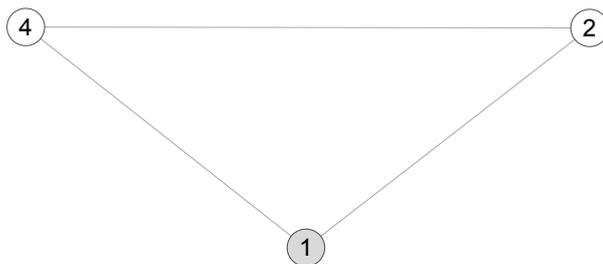
There are 7 2-D families of subalgebras to be analyzed.

Done.

There are 2 optimal families of 2-dimensional Lie subalgebras.

$\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_4\}$

$\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 3 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_4\}\}$



There are 1 3-D families of subalgebras to be analyzed.

Done.

There are 1 optimal families of 3-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}\}$

Time of computation: 595.903093

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Algebra n. 30

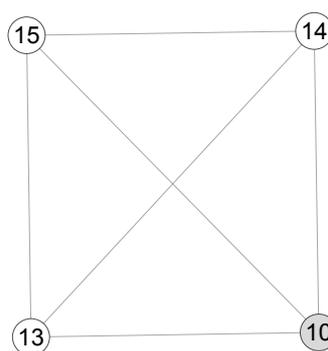
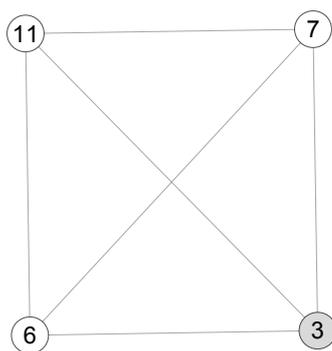
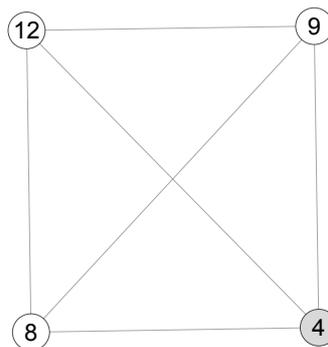
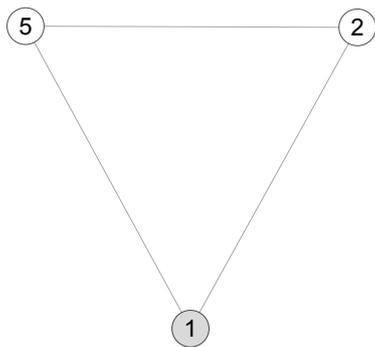
$$\begin{pmatrix} 0 & 0 & \mathfrak{E}_1 & -\mathfrak{E}_2 \\ 0 & 0 & \mathfrak{E}_2 & \mathfrak{E}_1 \\ -\mathfrak{E}_1 & -\mathfrak{E}_2 & 0 & 0 \\ \mathfrak{E}_2 & -\mathfrak{E}_1 & 0 & 0 \end{pmatrix}$$

There are 15 1-D families of subalgebras to be analyzed.

Done.

There are 4 optimal families of 1-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1\}, \{\mathfrak{E}_3\}, \{\mathfrak{E}_4\}, \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1\}, 3 \rightarrow \{\mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_4\}, 10 \rightarrow \{\mathfrak{E}_3 + \mathfrak{a}_1 \mathfrak{E}_4\}\}$



There are 8 2-D families of subalgebras to be analyzed.

Done.

There are 3 optimal families of 2-dimensional Lie subalgebras.

$\{\{\mathfrak{E}_1, \mathfrak{E}_2\}, \{\mathfrak{E}_1, \mathfrak{E}_3\}, \{\mathfrak{E}_3, \mathfrak{E}_4\}\}$
 $\{1 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_2\}, 2 \rightarrow \{\mathfrak{E}_1, \mathfrak{E}_3\}, 4 \rightarrow \{\mathfrak{E}_3, \mathfrak{E}_4\}\}$


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$\{2, 12\}, \{3, 4\}, \{3, 8\}, \{5, 2\}, \{5, 8\}, \{6, 8\}, \{9, 2\}, \{9, 4\}, \{10, 4\},$
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 $\{11, 4\}, \{13, 2\}, \{5, 14\}, \{9, 14\}, \{11, 12\}, \{13, 6\}, \{13, 10\}, \{13, 14\},$
 $\{\{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{-1, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, -1, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
 $\{\{1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0\},$
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 $\{0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1\}\}, \{\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 4\},$
 $\{1, 6\}, \{1, 10\}, \{1, 12\}, \{3, 4\}, \{5, 2\}, \{1, 14\}, \{5, 6\}\},$
 $\{\{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{-1, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, -1, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 0, 1\}, \{0, 1, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
 $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 1, 1, 0, 1\},$
 $\{0, 0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 1, 1, 0, 1\},$
 $\{0, 0, 1, 0, 0, 1, 1, 0, 1\}, \{0, 0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 0, 1, 0, 0, 1, 1, 0, 1\}\},$
 $\{\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 8\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{9, 2\}, \{1, 14\}\},$
 $\{\{0, 0, 0, 2\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-2, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, -1, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
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 $\{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\},$
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 $\{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0\},$
 $\{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1\}\}, \{\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 8\},$
 $\{4, 8\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{9, 2\}, \{9, 4\}, \{1, 14\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, -1, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
 $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1\}, \{0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0\},$
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 $\{0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0\},$
 $\{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1\}\}, \{\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 8\},$
 $\{4, 8\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{9, 2\}, \{9, 4\}, \{1, 14\}\},$
 $\{\{0, 0, 0, \frac{1}{2}\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-\frac{1}{2}, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, -1, 0, 0\}\},$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{0, 0, 0, -\frac{1}{2}\right\}, \left\{0, 0, \frac{1}{2}, 0\right\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}},$$

$$\left\{ \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1\} \right\},$$

$$\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 8\}, \{4, 8\}, \{1, 6\}, \{1, 10\},$$

$$\{1, 12\}, \{6, 8\}, \{9, 2\}, \{9, 4\}, \{1, 14\}, \{9, 6\},$$

$$\left\{ \{0, 0, 0, 2\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-2, 0, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, -1, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}},$$

$$\left\{ \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1\}, \{0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1\} \right\},$$

$$\{1, 2\}, \{1, 4\}, \{1, 8\}, \{2, 8\}, \{4, 8\}, \{1, 6\}, \{1, 10\},$$

$$\{1, 12\}, \{3, 8\}, \{9, 2\}, \{9, 4\}, \{1, 14\}, \{9, 10\},$$

$$\left\{ \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, -1, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}},$$

$$\left\{ \{1, 1, 0, 1, 0, 0, 0\}, \{1, 1, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 1, 1, 1\}, \{1, 1, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 1, 1, 1\}, \{0, 0, 1, 0, 1, 1, 1\} \right\},$$

$$\{1, 2\}, \{1, 4\}, \{1, 8\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{1, 14\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 0, 0\}, \{0, 1, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}},$$

$$\left\{ \{1, 1, 0, 1, 0, 0, 0\}, \{1, 1, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 1, 1, 1\}, \{1, 1, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 1, 1, 1\}, \{0, 0, 1, 0, 1, 1, 1\} \right\},$$

$$\{1, 2\}, \{1, 4\}, \{1, 8\}, \{1, 6\}, \{1, 10\}, \{1, 12\}, \{1, 14\},$$

$$\left\{ \{0, 0, 0, 2\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-2, 0, 0, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, -1, -1, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 0, 1\}, \{0, 1, -1, 0\} \right\},$$

$$\left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}},$$

$$\left\{ \{1, 0, 0, 0, 0, 0, 0\}, \{0, 1, 1, 0, 1, 1, 1, 1\}, \{0, 1, 1, 0, 1, 1, 1, 1\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 1, 1, 0, 1, 1, 1, 1\}, \{0, 1, 1, 0, 1, 1, 1, 1\} \right\},$$

```
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 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

In[98]:= **allalg3**

Out[98]=

```
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{{1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {2, 4, 8}, {1, 2, 12}, {1, 6, 8},
 {1, 10, 4}, {3, 4, 8}, {5, 2, 8}, {9, 2, 4}, {1, 10, 12},
 {5, 6, 8}, {9, 2, 12}, {9, 10, 4}, {9, 10, 12}},
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{{1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {2, 4, 8}, {1, 2, 12},
 {3, 4, 8}, {5, 2, 8}, {9, 2, 4}, {9, 2, 12}},
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 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}}}},
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 {0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}},
```


$\{\{0, 0, -2, 0\}, \{0, 0, 0, 0\}, \{2, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
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 $\{\{0, 0, -1, 0\}, \{0, 0, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
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 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
 $\{\{1, 0, 0, 0, 0\}, \{0, 1, 0, 1, 0\}, \{0, 0, 1, 0, 1\}, \{0, 1, 0, 1, 0\}, \{0, 0, 1, 0, 1\}\},$
 $\{\{1, 2, 4\}, \{1, 2, 8\}, \{2, 4, 8\}, \{1, 2, 12\}, \{9, 2, 4\}\},$
 $\{\{0, 0, 0, -2\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{2, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, -1, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
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 $\{\{1, 2, 4\}, \{1, 2, 8\}, \{2, 4, 8\}, \{1, 2, 12\}, \{5, 2, 8\}, \{9, 2, 4\}, \{9, 2, 12\}\},$
 $\{\{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{-1, 0, 0, 0\}\},$
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 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
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 $\{\{1, 2, 4\}, \{1, 2, 8\}, \{2, 4, 8\}, \{1, 2, 12\}, \{9, 2, 4\}\},$
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 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
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 $\{\{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{-1, -1, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 1\}, \{0, -1, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
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 $1, 0, 0, 1\}, \{0, 1, 0, 0, 1, 0, 0\}, \{0, 0, 1, 0, 0, 1, 0\}, \{0, 0, 0, 1, 0, 0, 1\}\},$
 $\{\{1, 2, 4\}, \{1, 2, 8\}, \{1, 4, 8\}, \{2, 4, 8\}, \{1, 2, 12\}, \{1, 10, 4\}, \{9, 2, 4\}\},$
 $\{\{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{-1, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 0, 0\}, \{0, 1, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, -\frac{1}{2}\}, \{0, 0, \frac{1}{2}, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$
 $\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0, 0\},$
 $\{0, 0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 1, 0, 0, 1\}\},$


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{{1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {1, 2, 12}, {1, 10, 4}},
{{{0, 0, 0, 1/2}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1/2, 0, 0, 0}},
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 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}},
{{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 1}, {0, 0, 0, 1, 0}, {0, 0, 1, 0, 1}},
 {{1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {1, 2, 12}, {1, 10, 4}},
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 {{0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}},
{{{1}}, {{1, 2, 4}}, {{{0, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 0, 0}, {0, 1, 0, 0}},
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In[99]:= SessionTime[]

Out[99]=

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