```
In[1]:=  (* Algebra of 2x2 traceless matrices *)

In[2]:=  SetDirectory[NotebookDirectory[]];

In[3]:=  << "SymboLie.wl"

         SymboLie (v. 1.6)– A Package for determining Optimal Systems of Lie Subalgebras.

In[4]:=  e1 = {{1, 0}, {0, -1}};

In[5]:=  e2 = {{0, 1}, {0, 0}};

In[6]:=  e3 = {{0, 0}, {1, 0}};

In[7]:=  cs = StructureConstants[{e1, e2, e3}, {}]

Out[7]= {{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
         {{0, 2, 0}, {-2, 0, 0}, {0, 0, 0}}, {{0, 0, -2}, {0, 0, 0}, {2, 0, 0}}}

In[8]:=  ct = CommutatorTable[cs]; ct // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 2\,\Xi_2 & -2\,\Xi_3 \\ -2\,\Xi_2 & 0 & \Xi_1 \\ 2\,\Xi_3 & -\Xi_1 & 0 \end{pmatrix}$$

```
In[9]:=  Timing[alg1 = SubAlgebra[cs, {{}, {}}, 1];]

         There are 7 1-D families of subalgebras to be analyzed.

         Done.

Out[9]= {0.491847, Null}

In[10]:= Timing[alg2 = SubAlgebra[cs, {{}, {}}, 2];]

         There are 2 2-D families of subalgebras to be analyzed.

         Done.

Out[10]=
         {0.313494, Null}

In[11]:= PrintOptimal[alg1]

         There are 3 optimal families of 1-dimensional Lie subalgebras.

Out[11]=
```
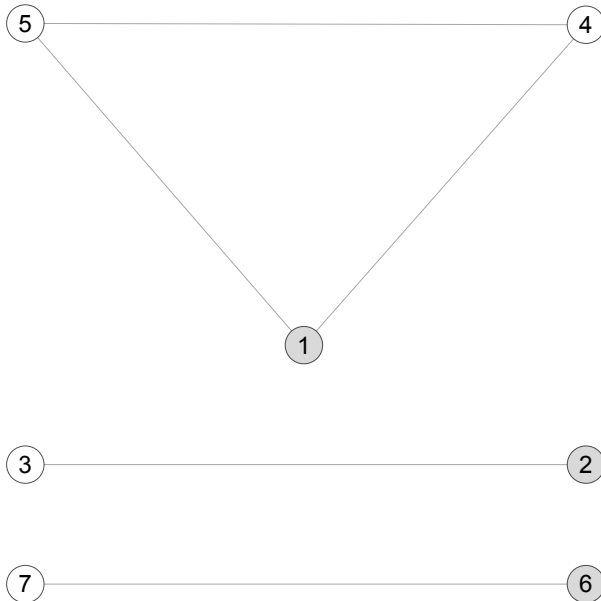$$\left\{ \{\Xi_1\}, \{\Xi_2\}, \left\{\Xi_2 + \alpha_1\,\Xi_3\right\} \right\}$$

In[12]:= `PrintGraph[alg1, 1]`

$\{1 \to \{\Xi_1\}, 2 \to \{\Xi_2\}, 3 \to \{\Xi_3\}, 4 \to \{\Xi_1 + \alpha_1 \Xi_2\},$
$5 \to \{\Xi_1 + \alpha_1 \Xi_3\}, 6 \to \{\Xi_2 + \alpha_1 \Xi_3\}, 7 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3\}\}$

Out[12]=

In[13]:= `PrintOptimal[alg2]`

There are 1 optimal families of 2-dimensional Lie subalgebras.

Out[13]=

$\{\{\Xi_1, \Xi_2\}\}$

In[14]:= `PrintGraph[alg2, 1]`

$\{1 \to \{\Xi_1, \Xi_2\}, 2 \to \{\Xi_1, \Xi_3\}\}$

Out[14]=

In[15]:= `alg = {alg1, alg2}`

Out[15]=

$\{\{\{\{1, 0, 0, 1, 1, 0, 0\},$
$\{0, 1, 1, 0, 0, 0, 0\}, \{0, 1, 1, 0, 0, 0, 0\}, \{1, 0, 0, 1, 1, 0, 0\},$
$\{1, 0, 0, 1, 1, 0, 0\}, \{0, 0, 0, 0, 0, 1, 1\}, \{0, 0, 0, 0, 0, 1, 1\}\},$
$\{\{1\}, \{2\}, \{4\}, \{3\}, \{5\}, \{6\}, \{7\}\}, \{\{\{0, 0, 0\}, \{0, 0, 1\}, \{0, -1, 0\}\},$
$\{\{0, 2, 0\}, \{-2, 0, 0\}, \{0, 0, 0\}\}, \{\{0, 0, -2\}, \{0, 0, 0\}, \{2, 0, 0\}\}\}\},$
$\{\{\{1, 1\}, \{1, 1\}\}, \{\{1, 2\}, \{1, 4\}\}, \{\{\{0, 0, 0\}, \{0, 0, 1\}, \{0, -1, 0\}\},$
$\{\{0, 2, 0\}, \{-2, 0, 0\}, \{0, 0, 0\}\}, \{\{0, 0, -2\}, \{0, 0, 0\}, \{2, 0, 0\}\}\}\}\}$

In[16]:= `SessionTime[]`

Out[16]=

4.875243