Algebra of Symmetries of 2 D stable viscous gas equations:
S . V . Meleshko. Group classification of two - dimensional stable viscous gas equations .
Int . J . Non - Linear Mech ., vol. 34, 1999, 449 - 456.

In[1]:=
```
SetDirectory[NotebookDirectory[]];
```

In[2]:=
```
<< "SymboLie.wl"
```

SymboLie (v. 1.6) – A Package for determining Optimal Systems of Lie Subalgebras.

In[3]:=
```
gens = {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0},
   {-y, x, -v, u, 0, 0}, {-x, -y, 0, 0, ρ, p}, {0, 0, u, v, ρ, 3 p}};
vars = {x, y, u, v, ρ, p}; pars = {{}, {}};
```

In[4]:=
```
cs = StructureConstants[gens, vars];
```

In[5]:=
```
ct = CommutatorTable[cs]; ct // MatrixForm
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \Xi_2 & -\Xi_1 & 0 \\ 0 & 0 & -\Xi_1 & -\Xi_2 & 0 \\ -\Xi_2 & \Xi_1 & 0 & 0 & 0 \\ \Xi_1 & \Xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[6]:=
```
Timing[alg1 = SubAlgebra[cs, pars, 1];]
```

There are 31 1–D families of subalgebras to be analyzed.

Done.

Out[6]= {8.92504, Null}

In[7]:=
```
Timing[alg2 = SubAlgebra[cs, pars, 2];]
```

There are 34 2–D families of subalgebras to be analyzed.

Done.

Out[7]= {32.7189, Null}

In[8]:=
```
Timing[alg3 = SubAlgebra[cs, pars, 3];]
```

There are 14 3–D families of subalgebras to be analyzed.

Done.

Out[8]= {25.2603, Null}

In[9]:=
```
Timing[alg4 = SubAlgebra[cs, pars, 4];]
```

There are 7 4–D families of subalgebras to be analyzed.

Done.

Out[9]= {7.53735, Null}

In[10]:=
```
PrintOptimal[alg1]
```

There are 9 optimal families of 1–dimensional Lie subalgebras.

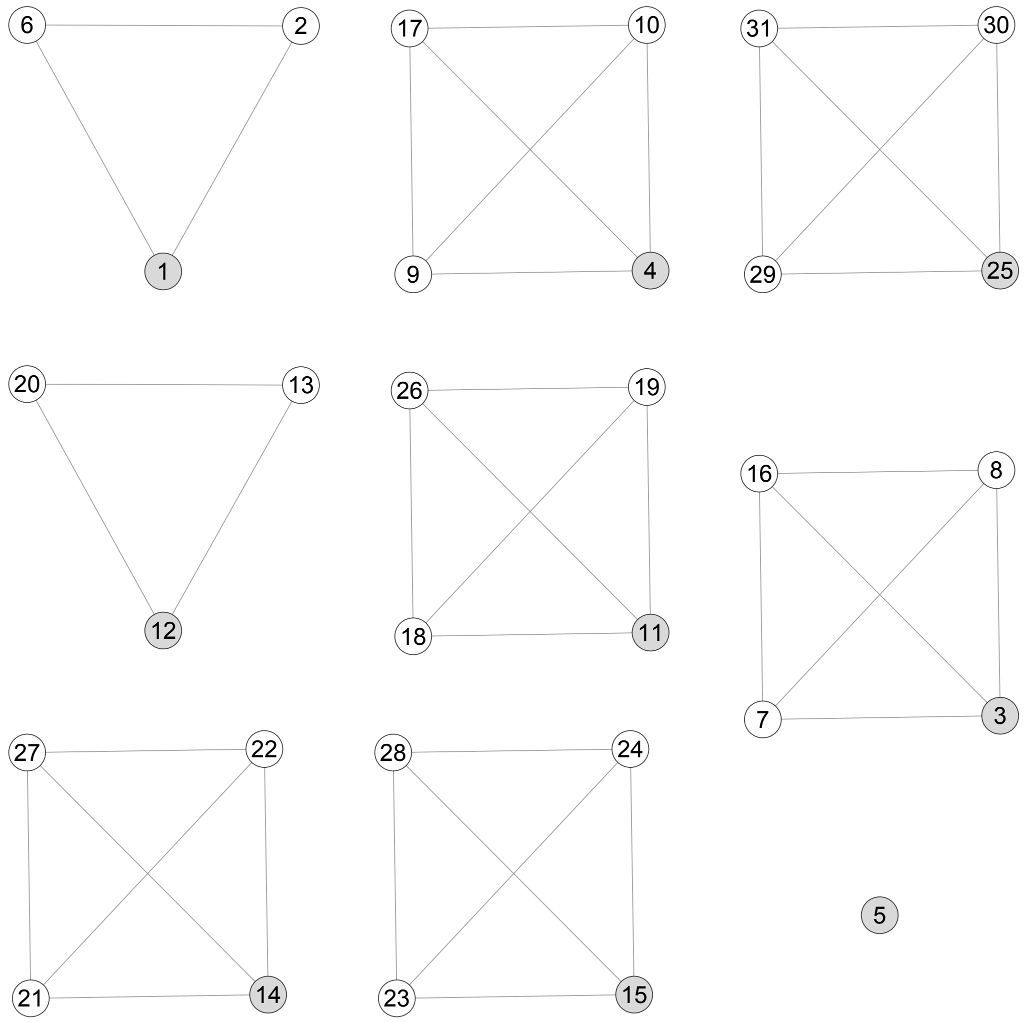
Out[10]=

$$\{\{\Xi_1\}, \{\Xi_3\}, \{\Xi_4\}, \{\Xi_5\}, \{\Xi_3 + a_1 \Xi_4\}, \{\Xi_1 + \alpha_1 \Xi_5\}, \{\Xi_3 + a_1 \Xi_5\}, \{\Xi_4 + a_1 \Xi_5\}, \{\Xi_3 + a_1 \Xi_4 + a_2 \Xi_5\}\}$$

In[11]:=
```
PrintGraph[alg1, 1]
```

{1 → $\{\Xi_1\}$, 2 → $\{\Xi_2\}$, 3 → $\{\Xi_3\}$, 4 → $\{\Xi_4\}$, 5 → $\{\Xi_5\}$, 6 → $\{\Xi_1 + \alpha_1 \Xi_2\}$, 7 → $\{\Xi_1 + \alpha_1 \Xi_3\}$,
8 → $\{\Xi_2 + \alpha_1 \Xi_3\}$, 9 → $\{\Xi_1 + \alpha_1 \Xi_4\}$, 10 → $\{\Xi_2 + \alpha_1 \Xi_4\}$, 11 → $\{\Xi_3 + a_1 \Xi_4\}$, 12 → $\{\Xi_1 + \alpha_1 \Xi_5\}$,
13 → $\{\Xi_2 + \alpha_1 \Xi_5\}$, 14 → $\{\Xi_3 + a_1 \Xi_5\}$, 15 → $\{\Xi_4 + a_1 \Xi_5\}$, 16 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3\}$,
17 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_4\}$, 18 → $\{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_4\}$, 19 → $\{\Xi_2 + \alpha_1 \Xi_3 + a_1 \Xi_4\}$, 20 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_5\}$,
21 → $\{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_5\}$, 22 → $\{\Xi_2 + \alpha_1 \Xi_3 + \alpha_2 \Xi_5\}$, 23 → $\{\Xi_1 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5\}$,
24 → $\{\Xi_2 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5\}$, 25 → $\{\Xi_3 + a_1 \Xi_4 + a_2 \Xi_5\}$, 26 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_4\}$,
27 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_5\}$, 28 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_4 + \alpha_3 \Xi_5\}$, 29 → $\{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_4 + a_2 \Xi_5\}$,
30 → $\{\Xi_2 + \alpha_1 \Xi_3 + a_1 \Xi_4 + a_2 \Xi_5\}$, 31 → $\{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_4 + \alpha_4 \Xi_5\}$}

Out[11]=



In[12]:=
```
PrintOptimal[alg2]
```

There are 12 optimal families of 2-dimensional Lie subalgebras.

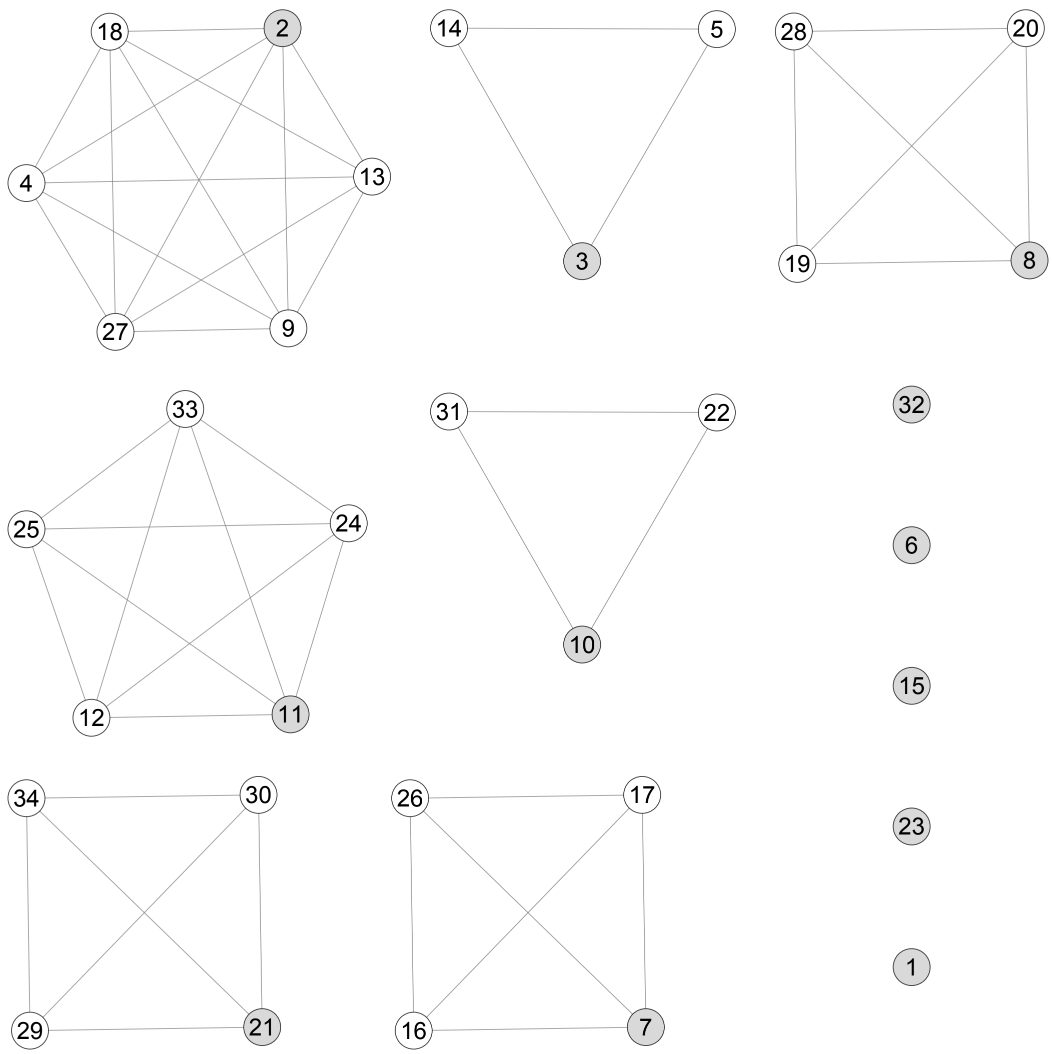
Out[12]=

$\{\{\Xi_1, \Xi_2\}, \{\Xi_1, \Xi_4\}, \{\Xi_1, \Xi_5\}, \{\Xi_3, \Xi_4\}, \{\Xi_3, \Xi_5\}, \{\Xi_4, \Xi_5\}, \{\Xi_1, \Xi_2 + \alpha_1 \Xi_5\},$
$\{\Xi_1, \Xi_4 + a_1 \Xi_5\}, \{\Xi_3, \Xi_4 + a_1 \Xi_5\}, \{\Xi_3 + a_1 \Xi_4, \Xi_5\}, \{\Xi_3 + a_1 \Xi_5, \Xi_4\}, \{\Xi_3 + a_1 \Xi_5, \Xi_4 + a_2 \Xi_5\}\}$

In[13]:= **PrintGraph[alg2, 1]**

$\{1 \to \{\Xi_1, \Xi_2\}, 2 \to \{\Xi_1, \Xi_4\}, 3 \to \{\Xi_1, \Xi_5\}, 4 \to \{\Xi_2, \Xi_4\}, 5 \to \{\Xi_2, \Xi_5\},$
$6 \to \{\Xi_3, \Xi_4\}, 7 \to \{\Xi_3, \Xi_5\}, 8 \to \{\Xi_4, \Xi_5\}, 9 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_4\}, 10 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_5\},$
$11 \to \{\Xi_1, \Xi_4 + a_1 \Xi_5\}, 12 \to \{\Xi_2, \Xi_4 + a_1 \Xi_5\}, 13 \to \{\Xi_1 + \alpha_1 \Xi_2, \Xi_4\}, 14 \to \{\Xi_1 + \alpha_1 \Xi_2, \Xi_5\},$
$15 \to \{\Xi_3, \Xi_4 + a_1 \Xi_5\}, 16 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_5\}, 17 \to \{\Xi_2 + \alpha_1 \Xi_3, \Xi_5\}, 18 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2\},$
$19 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_5\}, 20 \to \{\Xi_2 + \alpha_1 \Xi_4, \Xi_5\}, 21 \to \{\Xi_3 + a_1 \Xi_4, \Xi_5\}, 22 \to \{\Xi_1 + \alpha_1 \Xi_5, \Xi_2\},$
$23 \to \{\Xi_3 + a_1 \Xi_5, \Xi_4\}, 24 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5\}, 25 \to \{\Xi_1 + \alpha_1 \Xi_2, \Xi_4 + a_1 \Xi_5\},$
$26 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3, \Xi_5\}, 27 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2 + \alpha_2 \Xi_4\}, 28 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_4, \Xi_5\},$
$29 \to \{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_4, \Xi_5\}, 30 \to \{\Xi_2 + \alpha_1 \Xi_3 + a_1 \Xi_4, \Xi_5\}, 31 \to \{\Xi_1 + \alpha_1 \Xi_5, \Xi_2 + \alpha_2 \Xi_5\},$
$32 \to \{\Xi_3 + a_1 \Xi_5, \Xi_4 + a_2 \Xi_5\}, 33 \to \{\Xi_1 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5, \Xi_2\}, 34 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_4, \Xi_5\}\}$

Out[13]=



In[14]:= **PrintOptimal[alg3]**

There are 9 optimal families of 3-dimensional Lie subalgebras.

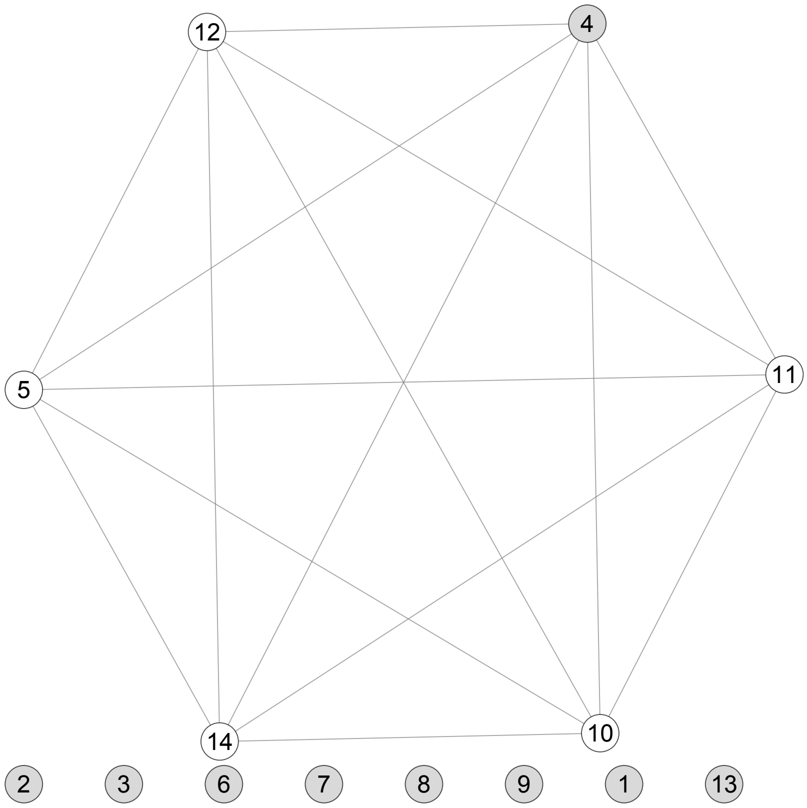
Out[14]=

$\{\{\Xi_1, \Xi_2, \Xi_3\}, \{\Xi_1, \Xi_2, \Xi_4\}, \{\Xi_1, \Xi_2, \Xi_5\}, \{\Xi_1, \Xi_4, \Xi_5\}, \{\Xi_3, \Xi_4, \Xi_5\},$
$\{\Xi_1, \Xi_2, \Xi_3 + a_1 \Xi_4\}, \{\Xi_1, \Xi_2, \Xi_3 + a_1 \Xi_5\}, \{\Xi_1, \Xi_2, \Xi_4 + a_1 \Xi_5\}, \{\Xi_1, \Xi_2, \Xi_3 + a_1 \Xi_4 + a_2 \Xi_5\}\}$

In[15]:= **PrintGraph[alg3, 1]**

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3\}, 2 \to \{\Xi_1, \Xi_2, \Xi_4\}, 3 \to \{\Xi_1, \Xi_2, \Xi_5\}, 4 \to \{\Xi_1, \Xi_4, \Xi_5\},$
$5 \to \{\Xi_2, \Xi_4, \Xi_5\}, 6 \to \{\Xi_3, \Xi_4, \Xi_5\}, 7 \to \{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_4\}, 8 \to \{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_5\},$
$9 \to \{\Xi_1, \Xi_2, \Xi_4 + \underset{.}{a}_1 \Xi_5\}, 10 \to \{\Xi_1, \Xi_2 + \underset{.}{\alpha}_1 \Xi_4, \Xi_5\}, 11 \to \{\Xi_1 + \underset{.}{\alpha}_1 \Xi_2, \Xi_4, \Xi_5\},$
$12 \to \{\Xi_1 + \underset{.}{\alpha}_1 \Xi_4, \Xi_2, \Xi_5\}, 13 \to \{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_4 + \underset{.}{a}_2 \Xi_5\}, 14 \to \{\Xi_1 + \underset{.}{\alpha}_1 \Xi_4, \Xi_2 + \underset{.}{\alpha}_2 \Xi_4, \Xi_5\}\}$

Out[15]=



In[16]:= **PrintOptimal[alg4]**

There are 7 optimal families of 4-dimensional Lie subalgebras.

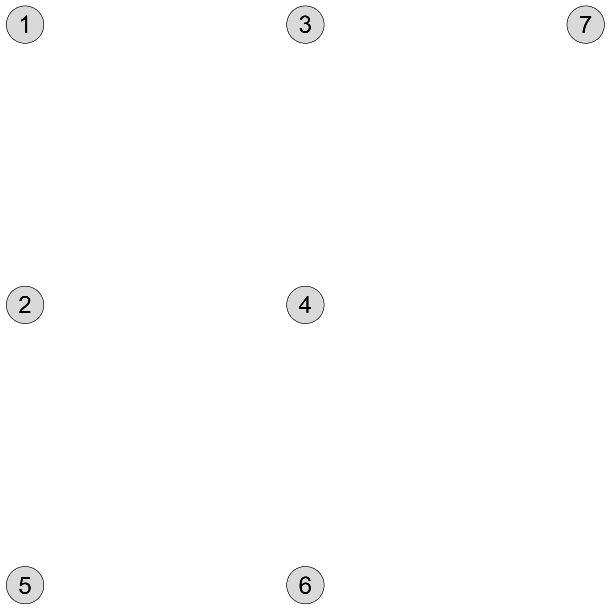
Out[16]=

$\{\{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}, \{\Xi_1, \Xi_2, \Xi_3, \Xi_5\}, \{\Xi_1, \Xi_2, \Xi_4, \Xi_5\}, \{\Xi_1, \Xi_2, \Xi_3, \Xi_4 + \underset{.}{a}_1 \Xi_5\},$
$\{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_4, \Xi_5\}, \{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_5, \Xi_4\}, \{\Xi_1, \Xi_2, \Xi_3 + \underset{.}{a}_1 \Xi_5, \Xi_4 + \underset{.}{a}_2 \Xi_5\}\}$

In[17]:=
```
PrintGraph[alg4, 1]
```

{1 → {Ξ1, Ξ2, Ξ3, Ξ4}, 2 → {Ξ1, Ξ2, Ξ3, Ξ5}, 3 → {Ξ1, Ξ2, Ξ4, Ξ5}, 4 → {Ξ1, Ξ2, Ξ3, Ξ4 + a1 Ξ5},
5 → {Ξ1, Ξ2, Ξ3 + a1 Ξ4, Ξ5}, 6 → {Ξ1, Ξ2, Ξ3 + a1 Ξ5, Ξ4}, 7 → {Ξ1, Ξ2, Ξ3 + a1 Ξ5, Ξ4 + a2 Ξ5}}

Out[17]=



In[18]:=
```
alg = {alg1, alg2, alg3, alg4}
```

Out[18]=

```
{{{{1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 1, 0, 0, 0, 1, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 1,
    0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,
    0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1,
    0, 0, 0, 1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0,
    1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0,
    0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
```

```
0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1,
0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1}, {0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 1, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1}},
{{1}, {2}, {4}, {8}, {16}, {3}, {5}, {6}, {9}, {10}, {12}, {17},
{18}, {20}, {24}, {7}, {11}, {13}, {14}, {19}, {21}, {22},
{25}, {26}, {28}, {15}, {23}, {27}, {29}, {30}, {31}},
{{{0, 0, 0, -1, 0}, {0, 0, -1, 0, 0}, {0, 1, 0, 0, 0}, {1, 0, 0, 0, 0},
{0, 0, 0, 0, 0}}, {{0, 0, 1, 0, 0}, {0, 0, 0, -1, 0},
{-1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}},
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
1, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0},
{0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0,
```

```
 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0,
 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0,
 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0,
 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
 0, 0, 0, 0, 1, 1, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
 1, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
 0, 0, 0, 0, 1, 1, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
 1, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1}},
{{1, 2}, {1, 8}, {1, 16}, {2, 8}, {2, 16}, {4, 8}, {4, 16}, {8, 16},
 {1, 10}, {1, 18}, {1, 24}, {2, 24}, {3, 8}, {3, 16}, {4, 24},
 {5, 16}, {6, 16}, {9, 2}, {9, 16}, {10, 16}, {12, 16},
 {17, 2}, {20, 8}, {1, 26}, {3, 24}, {7, 16}, {9, 10}, {11, 16},
 {13, 16}, {14, 16}, {17, 18}, {20, 24}, {25, 2}, {15, 16}},
{{{0, 0, 0, -1, 0}, {0, 0, -1, 0, 0}, {0, 1, 0, 0, 0}, {1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0}}, {{0, 0, 1, 0, 0}, {0, 0, 0, -1, 0},
  {-1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}},
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1},
 {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1},
 {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1}, {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1}},
{{1, 2, 4}, {1, 2, 8}, {1, 2, 16}, {1, 8, 16}, {2, 8, 16},
 {4, 8, 16}, {1, 2, 12}, {1, 2, 20}, {1, 2, 24}, {1, 10, 16},
 {3, 8, 16}, {9, 2, 16}, {1, 2, 28}, {9, 10, 16}},
{{{0, 0, 0, -1, 0}, {0, 0, -1, 0, 0}, {0, 1, 0, 0, 0}, {1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0}}, {{0, 0, 1, 0, 0}, {0, 0, 0, -1, 0},
  {-1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}},
{{{1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}},
 {{1, 2, 4, 8}, {1, 2, 4, 16}, {1, 2, 8, 16}, {1, 2, 4, 24},
  {1, 2, 12, 16}, {1, 2, 20, 8}, {1, 2, 20, 24}},
 {{{0, 0, 0, -1, 0}, {0, 0, -1, 0, 0}, {0, 1, 0, 0, 0}, {1, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}}, {{0, 0, 1, 0, 0}, {0, 0, 0, -1, 0},
   {-1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}}}
```

In[19]:=
```
SessionTime[]
```

Out[19]=
```
83.031422
```