Algebra of Lie symmetries of KdV equation

In[1]:=
```
SetDirectory[NotebookDirectory[]];
```

In[2]:=
```
<< "SymboLie.wl"
```

SymboLie (v. 1.6) - A Package for determining Optimal Systems of Lie Subalgebras.

In[3]:=
```
gens = {{1, 0, 0}, {0, 1, 0}, {0, t, 1}, {3 t, x, -2 u}};
vars = {t, x, u};
pars = {{}, {}};
```

In[4]:=
```
cs = StructureConstants[gens, vars];
```

In[5]:=
```
CommutatorTable[cs] // MatrixForm
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \Xi_2 & 3\,\Xi_1 \\ 0 & 0 & 0 & \Xi_2 \\ -\Xi_2 & 0 & 0 & -2\,\Xi_3 \\ -3\,\Xi_1 & -\Xi_2 & 2\,\Xi_3 & 0 \end{pmatrix}$$

In[6]:=
```
alg1 = SubAlgebra[cs, pars, 1];
```

There are 15 1-D families of subalgebras to be analyzed.

Done.

In[7]:=
```
alg2 = SubAlgebra[cs, pars, 2];
```

There are 11 2-D families of subalgebras to be analyzed.

Done.

In[8]:=
```
alg3 = SubAlgebra[cs, pars, 3];
```

There are 5 3-D families of subalgebras to be analyzed.

Done.

In[9]:=
```
PrintOptimal[alg1]
```

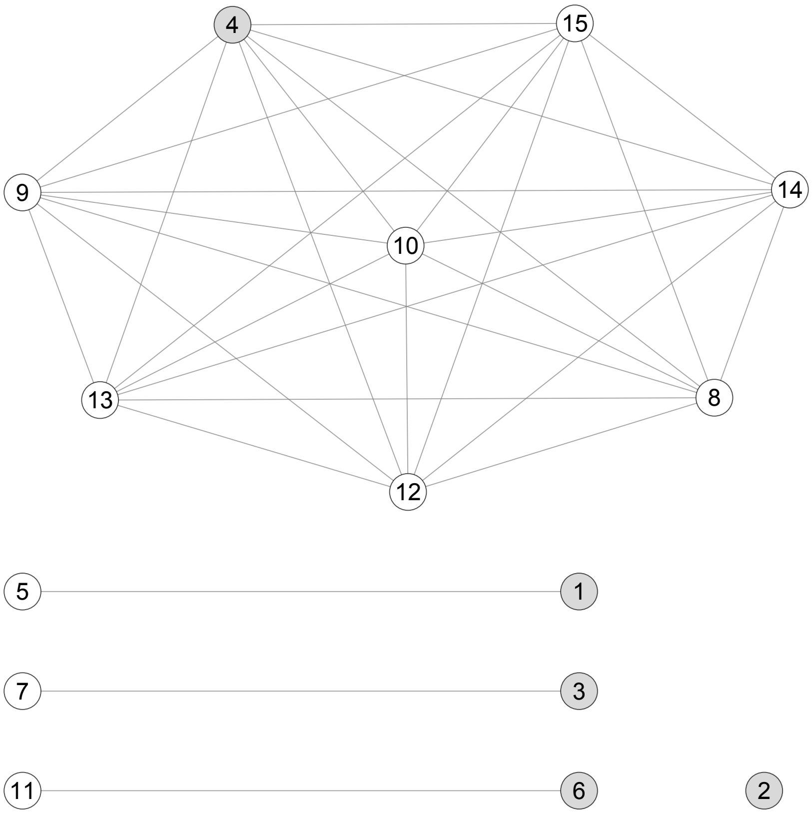
There are 5 optimal families of 1-dimensional Lie subalgebras.

Out[9]= $\{\{\Xi_1\}, \{\Xi_2\}, \{\Xi_3\}, \{\Xi_4\}, \{\Xi_1 + \underset{\cdot}{\alpha}_1\,\Xi_3\}\}$

In[10]:= `PrintGraph[alg1, 1]`

$\{1 \to \{\Xi_1\},\ 2 \to \{\Xi_2\},\ 3 \to \{\Xi_3\},\ 4 \to \{\Xi_4\},\ 5 \to \{\Xi_1 + \alpha_1\, \Xi_2\},$
$6 \to \{\Xi_1 + \alpha_1\, \Xi_3\},\ 7 \to \{\Xi_2 + \alpha_1\, \Xi_3\},\ 8 \to \{\Xi_1 + \alpha_1\, \Xi_4\},\ 9 \to \{\Xi_2 + \alpha_1\, \Xi_4\},$
$10 \to \{\Xi_3 + \alpha_1\, \Xi_4\},\ 11 \to \{\Xi_1 + \alpha_1\, \Xi_2 + a_1\, \Xi_3\},\ 12 \to \{\Xi_1 + \alpha_1\, \Xi_2 + \alpha_2\, \Xi_4\},$
$13 \to \{\Xi_1 + \alpha_1\, \Xi_3 + a_1\, \Xi_4\},\ 14 \to \{\Xi_2 + \alpha_1\, \Xi_3 + \alpha_2\, \Xi_4\},\ 15 \to \{\Xi_1 + \alpha_1\, \Xi_2 + \alpha_2\, \Xi_3 + a_1\, \Xi_4\}\}$

Out[10]=



In[11]:= `PrintOptimal[alg2]`

There are 6 optimal families of 2-dimensional Lie subalgebras.

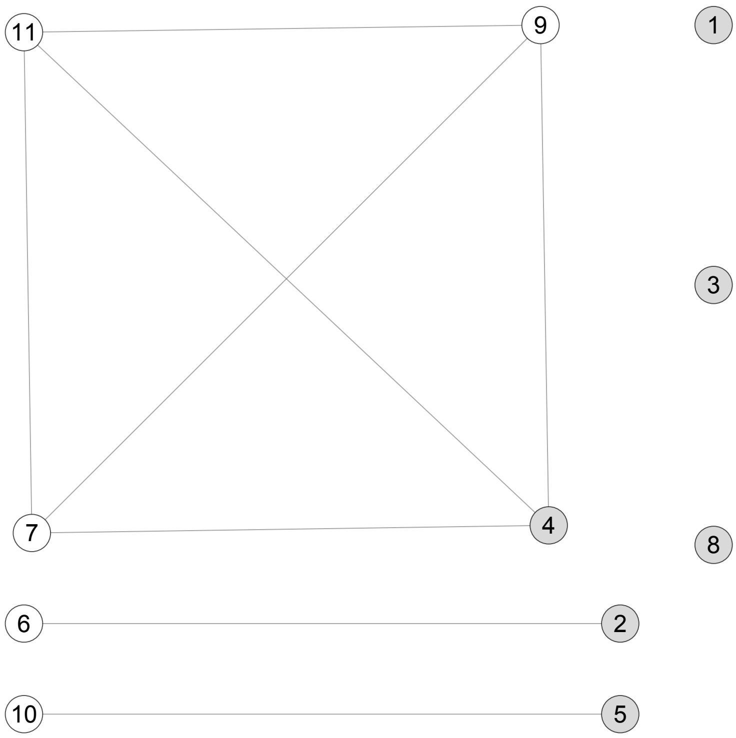
Out[11]=

$\{\{\Xi_1, \Xi_2\}, \{\Xi_1, \Xi_4\}, \{\Xi_2, \Xi_3\}, \{\Xi_2, \Xi_4\}, \{\Xi_3, \Xi_4\}, \{\Xi_1 + \alpha_1\, \Xi_3, \Xi_2\}\}$

In[12]:= **PrintGraph[alg2, 1]**

$\{1 \to \{\Xi_1, \Xi_2\}, 2 \to \{\Xi_1, \Xi_4\}, 3 \to \{\Xi_2, \Xi_3\}, 4 \to \{\Xi_2, \Xi_4\},$
$5 \to \{\Xi_3, \Xi_4\}, 6 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_4\}, 7 \to \{\Xi_2, \Xi_3 + \alpha_1 \Xi_4\}, 8 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_2\},$
$9 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2\}, 10 \to \{\Xi_2 + \alpha_1 \Xi_4, \Xi_3\}, 11 \to \{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_4, \Xi_2\}\}$

Out[12]=



In[13]:= **PrintOptimal[alg3]**

There are 3 optimal families of 3-dimensional Lie subalgebras.

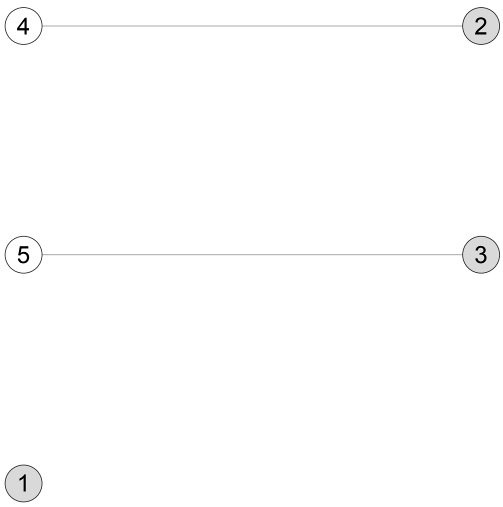
Out[13]=

$\{\{\Xi_1, \Xi_2, \Xi_3\}, \{\Xi_1, \Xi_2, \Xi_4\}, \{\Xi_2, \Xi_3, \Xi_4\}\}$

In[14]:= **PrintGraph[alg3, 1]**

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3\}, 2 \to \{\Xi_1, \Xi_2, \Xi_4\},$
$3 \to \{\Xi_2, \Xi_3, \Xi_4\}, 4 \to \{\Xi_1, \Xi_2, \Xi_3 + \alpha_1 \Xi_4\}, 5 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2, \Xi_3\}\}$

Out[14]=



In[15]:=
```
alg = {alg1, alg2, alg3}
```
Out[15]=
```
{{{{1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1,
    1, 1, 0, 1, 1, 1, 1}, {1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1},
   {0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1}, {0, 0, 0, 1, 0, 0, 0, 1,
    1, 1, 0, 1, 1, 1, 1}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1}, {0, 0, 0, 1, 0, 0, 0, 1,
    1, 1, 0, 1, 1, 1, 1}, {0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1},
   {0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1}},
  {{1}, {2}, {4}, {8}, {3}, {5}, {6}, {9}, {10}, {12}, {7}, {11}, {13}, {14}, {15}},
  {{{0, 0, 0, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-3, 0, 0, 0}},
   {{0, 0, 1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, -1, 0, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, -2}, {0, 0, 2, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}},
 {{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1},
   {0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0},
   {0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1}}, {{1, 2}, {1, 8}, {2, 4}, {2, 8},
   {4, 8}, {1, 10}, {2, 12}, {5, 2}, {9, 2}, {10, 4}, {13, 2}},
  {{{0, 0, 0, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-3, 0, 0, 0}},
   {{0, 0, 1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, -1, 0, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, -2}, {0, 0, 2, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}},
 {{{1, 0, 0, 0, 0}, {0, 1, 0, 1, 0}, {0, 0, 1, 0, 1}, {0, 1, 0, 1, 0}, {0, 0, 1, 0, 1}},
  {{1, 2, 4}, {1, 2, 8}, {2, 4, 8}, {1, 2, 12}, {9, 2, 4}},
  {{{0, 0, 0, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-3, 0, 0, 0}},
   {{0, 0, 1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, -1, 0, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, -2}, {0, 0, 2, 0}},
   {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}}
```

In[16]:=
```
SessionTime[]
```
Out[16]=
```
8.949198
```