Algebra of Lie symmetries of linear Heat equation .

Kai - Seng Chou, Guan - Xin Li, Changzheng Qu . A note on optimal systems for the heat equation .

Journal of Mathematical Analysis and Applicatons, 261, 741 - 751, 2001.

In[1]:=
```
SetDirectory[NotebookDirectory[]];
```

In[2]:=
```
<< "SymboLie.wl"
```

```
SymboLie (v. 1.6)- A Package for determining Optimal Systems of Lie Subalgebras.
```

In[3]:=
```
vars = {x, t, u};
gens = {{0, 1, 0}, {1, 0, 0}, {x, 2 t, 0},
  {0, 0, u}, {2 t, 0, -x u}, {4 t x, 4 t^2, -(x^2 + 2 t) u}};
pars = {{}, {}};
```

In[4]:=
```
cs = StructureConstants[gens, vars];
```

In[5]:=
```
ct = CommutatorTable[cs]; ct // MatrixForm
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 2\Xi_1 & 0 & 2\Xi_2 & 4\Xi_3 - 2\Xi_4 \\ 0 & 0 & \Xi_2 & 0 & -\Xi_4 & 2\Xi_5 \\ -2\Xi_1 & -\Xi_2 & 0 & 0 & \Xi_5 & 2\Xi_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -2\Xi_2 & \Xi_4 & -\Xi_5 & 0 & 0 & 0 \\ -4\Xi_3 + 2\Xi_4 & -2\Xi_5 & -2\Xi_6 & 0 & 0 & 0 \end{pmatrix}$$

In[6]:=
```
FastRun = 1;
```

In[7]:=
```
Timing[alg1 = SubAlgebra[cs, {{}, {}}, 1];]
```

```
There are 63 1-D families of subalgebras to be analyzed.
Done.
```

Out[7]= {1176.55, Null}

In[8]:=
```
PrintOptimal[alg1]
```

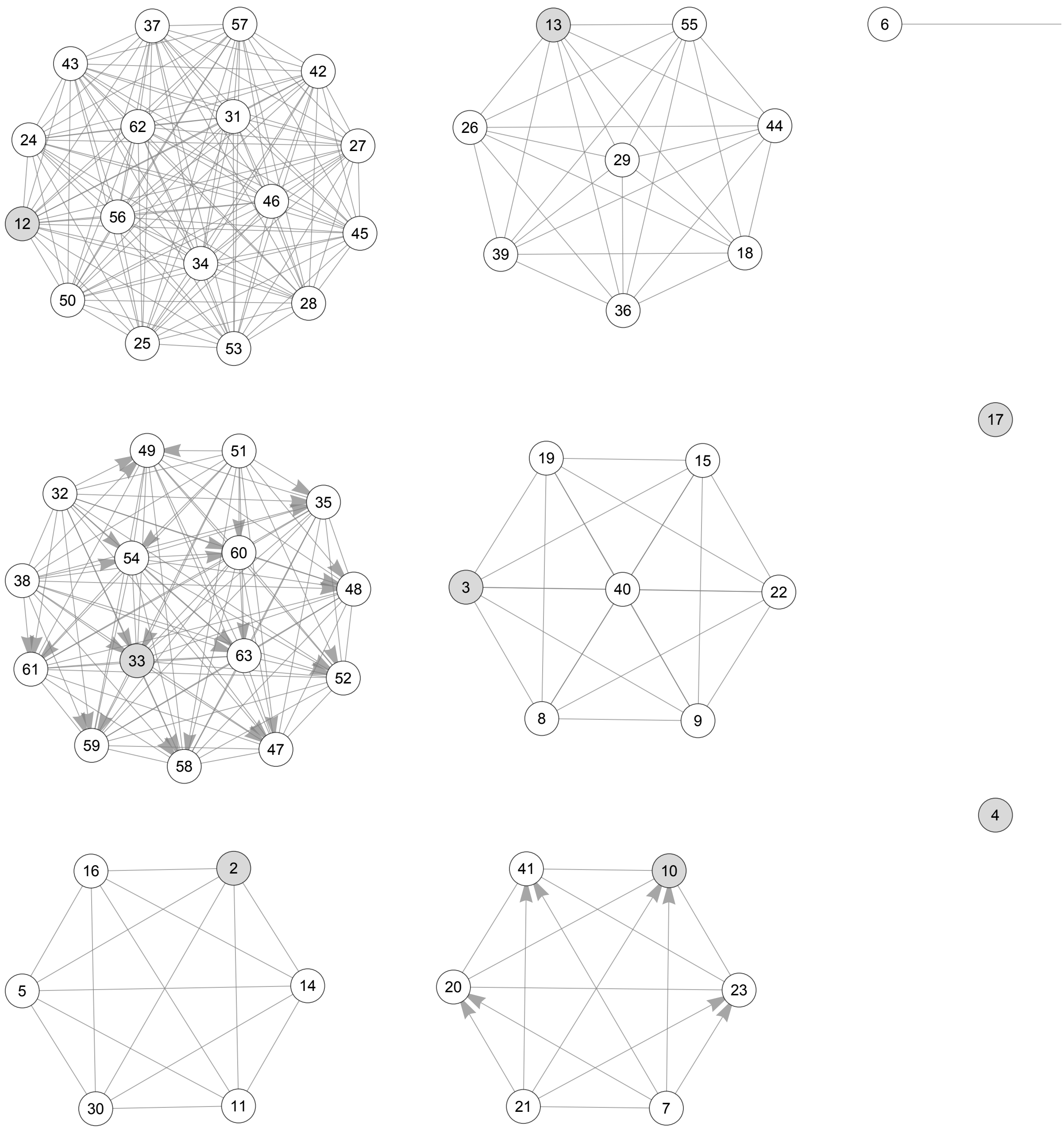
```
There are 9 optimal families of 1-dimensional Lie subalgebras.
```

Out[8]= $\{\{\Xi_1\}, \{\Xi_2\}, \{\Xi_3\}, \{\Xi_4\}, \{\Xi_1 + \alpha_1 \Xi_4\}, \{\Xi_3 + a_1 \Xi_4\}, \{\Xi_1 + \alpha_1 \Xi_5\}, \{\Xi_1 + \alpha_1 \Xi_6\}, \{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_6\}\}$

In[9]:=
```
PrintGraph[alg1, 1]
```

$\{1 \to \{\Xi_1\},\ 2 \to \{\Xi_2\},\ 3 \to \{\Xi_3\},\ 4 \to \{\Xi_4\},\ 5 \to \{\Xi_5\},\ 6 \to \{\Xi_6\},\ 7 \to \{\Xi_1 + \alpha_1\,\Xi_2\},\ 8 \to \{\Xi_1 + \alpha_1\,\Xi_3\},$
$9 \to \{\Xi_2 + \alpha_1\,\Xi_3\},\ 10 \to \{\Xi_1 + \alpha_1\,\Xi_4\},\ 11 \to \{\Xi_2 + \alpha_1\,\Xi_4\},\ 12 \to \{\Xi_3 + a_1\,\Xi_4\},\ 13 \to \{\Xi_1 + \alpha_1\,\Xi_5\},$
$14 \to \{\Xi_2 + \alpha_1\,\Xi_5\},\ 15 \to \{\Xi_3 + \alpha_1\,\Xi_5\},\ 16 \to \{\Xi_4 + \alpha_1\,\Xi_5\},\ 17 \to \{\Xi_1 + \alpha_1\,\Xi_6\},\ 18 \to \{\Xi_2 + \alpha_1\,\Xi_6\},$
$19 \to \{\Xi_3 + \alpha_1\,\Xi_6\},\ 20 \to \{\Xi_4 + \alpha_1\,\Xi_6\},\ 21 \to \{\Xi_5 + \alpha_1\,\Xi_6\},\ 22 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3\},$
$23 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4\},\ 24 \to \{\Xi_1 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4\},\ 25 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4\},\ 26 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_5\},$
$27 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_5\},\ 28 \to \{\Xi_2 + \alpha_1\,\Xi_3 + a_1\,\Xi_5\},\ 29 \to \{\Xi_1 + \alpha_1\,\Xi_4 + a_1\,\Xi_5\},\ 30 \to \{\Xi_2 + \alpha_1\,\Xi_4 + \alpha_2\,\Xi_5\},$
$31 \to \{\Xi_3 + a_1\,\Xi_4 + \alpha_1\,\Xi_5\},\ 32 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_6\},\ 33 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_6\},\ 34 \to \{\Xi_2 + \alpha_1\,\Xi_3 + a_1\,\Xi_6\},$
$35 \to \{\Xi_1 + \alpha_1\,\Xi_4 + a_1\,\Xi_6\},\ 36 \to \{\Xi_2 + \alpha_1\,\Xi_4 + a_1\,\Xi_6\},\ 37 \to \{\Xi_3 + a_1\,\Xi_4 + \alpha_1\,\Xi_6\},$
$38 \to \{\Xi_1 + \alpha_1\,\Xi_5 + a_1\,\Xi_6\},\ 39 \to \{\Xi_2 + \alpha_1\,\Xi_5 + a_1\,\Xi_6\},\ 40 \to \{\Xi_3 + \alpha_1\,\Xi_5 + \alpha_2\,\Xi_6\},$
$41 \to \{\Xi_4 + \alpha_1\,\Xi_5 + \alpha_2\,\Xi_6\},\ 42 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + a_1\,\Xi_4\},\ 43 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + a_1\,\Xi_5\},$
$44 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4 + a_1\,\Xi_5\},\ 45 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_4 + a_2\,\Xi_5\},\ 46 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + \alpha_3\,\Xi_5\},$
$47 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_3 + a_2\,\Xi_6\},\ 48 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4 + a_1\,\Xi_6\},\ 49 \to \{\Xi_1 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + \alpha_3\,\Xi_6\},$
$50 \to \{\Xi_2 + \alpha_1\,\Xi_3 + a_1\,\Xi_4 + a_2\,\Xi_6\},\ 51 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_5 + a_2\,\Xi_6\},\ 52 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_5 + a_2\,\Xi_6\},$
$53 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_5 + a_1\,\Xi_6\},\ 54 \to \{\Xi_1 + \alpha_1\,\Xi_4 + \alpha_2\,\Xi_5 + a_1\,\Xi_6\},\ 55 \to \{\Xi_2 + \alpha_1\,\Xi_4 + \alpha_2\,\Xi_5 + a_1\,\Xi_6\},$
$56 \to \{\Xi_3 + a_1\,\Xi_4 + \alpha_1\,\Xi_5 + \alpha_2\,\Xi_6\},\ 57 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + \alpha_3\,\Xi_4 + \alpha_4\,\Xi_5\},$
$58 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + a_1\,\Xi_4 + a_2\,\Xi_6\},\ 59 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_3 + a_2\,\Xi_5 + a_3\,\Xi_6\},$
$60 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4 + a_1\,\Xi_5 + a_2\,\Xi_6\},\ 61 \to \{\Xi_1 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + a_1\,\Xi_5 + a_2\,\Xi_6\},$
$62 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + \alpha_3\,\Xi_5 + \alpha_4\,\Xi_6\},\ 63 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + \alpha_3\,\Xi_4 + \alpha_4\,\Xi_5 + \alpha_5\,\Xi_6\}\}$

Out[9]=



In[10]:= **Timing[alg2 = SubAlgebra[cs, {{}, {}}, 2];]**

```
There are 55 2-D families of subalgebras to be analyzed.

Done.
```

Out[10]=

{2158.85, Null}

In[11]:= **PrintOptimal[alg2]**

```
There are 13 optimal families of 2-dimensional Lie subalgebras.
```

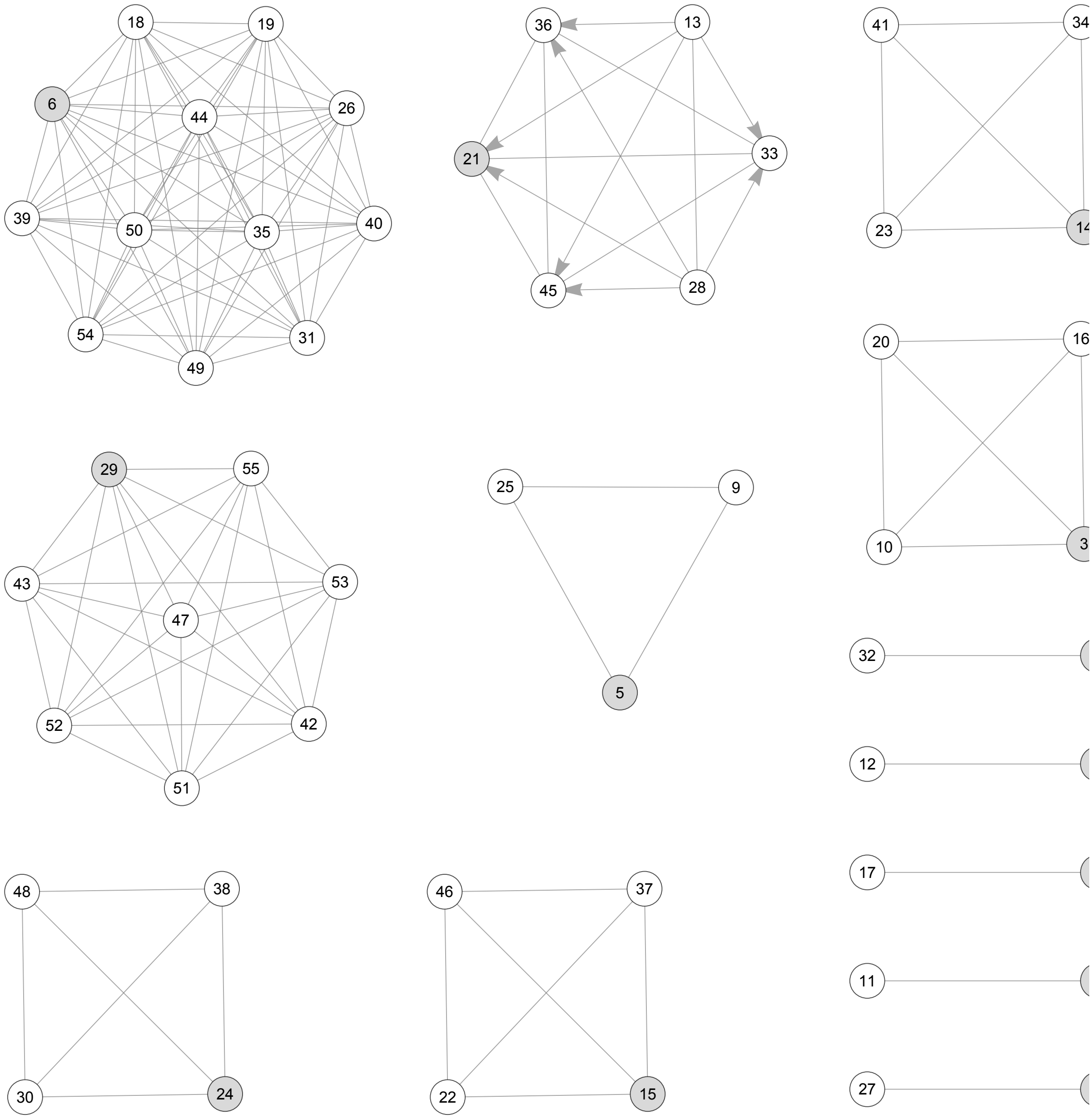
Out[11]=

$\{\{\Xi_1, \Xi_2\}, \{\Xi_1, \Xi_3\}, \{\Xi_1, \Xi_4\}, \{\Xi_2, \Xi_3\}, \{\Xi_2, \Xi_4\}, \{\Xi_3, \Xi_4\}, \{\Xi_3, \Xi_5\}, \{\Xi_3, \Xi_6\},$
$\{\Xi_1 + \underset{\cdot}{\alpha}_1 \Xi_4, \Xi_2\}, \{\Xi_1, \Xi_3 + \underset{\cdot}{a}_1 \Xi_4\}, \{\Xi_2, \Xi_3 + \underset{\cdot}{a}_1 \Xi_4\}, \{\Xi_1 + \underset{\cdot}{\alpha}_1 \Xi_5, \Xi_4\}, \{\Xi_1 + \underset{\cdot}{\alpha}_1 \Xi_6, \Xi_4\}\}$

In[12]:= **PrintGraph[alg2, 1]**

$\{1 \to \{\Xi_1, \Xi_2\}, 2 \to \{\Xi_1, \Xi_3\}, 3 \to \{\Xi_1, \Xi_4\}, 4 \to \{\Xi_2, \Xi_3\}, 5 \to \{\Xi_2, \Xi_4\},$
$6 \to \{\Xi_3, \Xi_4\}, 7 \to \{\Xi_3, \Xi_5\}, 8 \to \{\Xi_3, \Xi_6\}, 9 \to \{\Xi_4, \Xi_5\}, 10 \to \{\Xi_4, \Xi_6\}, 11 \to \{\Xi_5, \Xi_6\},$
$12 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_3\}, 13 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_4\}, 14 \to \{\Xi_1, \Xi_3 + a_1 \Xi_4\}, 15 \to \{\Xi_2, \Xi_3 + a_1 \Xi_4\},$
$16 \to \{\Xi_1 + \alpha_1 \Xi_2, \Xi_4\}, 17 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_2\}, 18 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_4\}, 19 \to \{\Xi_2 + \alpha_1 \Xi_3, \Xi_4\},$
$20 \to \{\Xi_4, \Xi_5 + \alpha_1 \Xi_6\}, 21 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2\}, 22 \to \{\Xi_3 + a_1 \Xi_4, \Xi_5\}, 23 \to \{\Xi_3 + a_1 \Xi_4, \Xi_6\},$
$24 \to \{\Xi_1 + \alpha_1 \Xi_5, \Xi_4\}, 25 \to \{\Xi_2 + \alpha_1 \Xi_5, \Xi_4\}, 26 \to \{\Xi_3 + \alpha_1 \Xi_5, \Xi_4\}, 27 \to \{\Xi_3 + \alpha_1 \Xi_5, \Xi_6\},$
$28 \to \{\Xi_4 + \alpha_1 \Xi_5, \Xi_6\}, 29 \to \{\Xi_1 + \alpha_1 \Xi_6, \Xi_4\}, 30 \to \{\Xi_2 + \alpha_1 \Xi_6, \Xi_4\}, 31 \to \{\Xi_3 + \alpha_1 \Xi_6, \Xi_4\},$
$32 \to \{\Xi_3 + \alpha_1 \Xi_6, \Xi_5\}, 33 \to \{\Xi_4 + \alpha_1 \Xi_6, \Xi_5\}, 34 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_3 + \alpha_2 \Xi_4\},$
$35 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3, \Xi_4\}, 36 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2 + \alpha_2 \Xi_4\}, 37 \to \{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_4, \Xi_2\},$
$38 \to \{\Xi_1 + \alpha_1 \Xi_2 + a_1 \Xi_5, \Xi_4\}, 39 \to \{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_5, \Xi_4\}, 40 \to \{\Xi_2 + \alpha_1 \Xi_3 + \alpha_2 \Xi_5, \Xi_4\},$
$41 \to \{\Xi_3 + a_1 \Xi_4 + \alpha_1 \Xi_5, \Xi_6\}, 42 \to \{\Xi_1 + \alpha_1 \Xi_2 + a_1 \Xi_6, \Xi_4\}, 43 \to \{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_6, \Xi_4\},$
$44 \to \{\Xi_2 + \alpha_1 \Xi_3 + a_1 \Xi_6, \Xi_4\}, 45 \to \{\Xi_4 + \alpha_1 \Xi_6, \Xi_5 + \alpha_2 \Xi_6\}, 46 \to \{\Xi_3 + a_1 \Xi_4 + \alpha_1 \Xi_6, \Xi_5\},$
$47 \to \{\Xi_1 + \alpha_1 \Xi_5 + a_1 \Xi_6, \Xi_4\}, 48 \to \{\Xi_2 + \alpha_1 \Xi_5 + a_1 \Xi_6, \Xi_4\}, 49 \to \{\Xi_3 + \alpha_1 \Xi_5 + \alpha_2 \Xi_6, \Xi_4\},$
$50 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_5, \Xi_4\}, 51 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + a_1 \Xi_6, \Xi_4\},$
$52 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_5 + a_1 \Xi_6, \Xi_4\}, 53 \to \{\Xi_1 + \alpha_1 \Xi_3 + a_1 \Xi_5 + \alpha_2 \Xi_6, \Xi_4\},$
$54 \to \{\Xi_2 + \alpha_1 \Xi_3 + \alpha_2 \Xi_5 + \alpha_3 \Xi_6, \Xi_4\}, 55 \to \{\Xi_1 + \alpha_1 \Xi_2 + \alpha_2 \Xi_3 + \alpha_3 \Xi_5 + \alpha_4 \Xi_6, \Xi_4\}\}$

Out[12]=



In[13]:=

```
Timing[alg3 = SubAlgebra[cs, {{}, {}}, 3];]
```

There are 21 3-D families of subalgebras to be analyzed.

Done.

Out[13]=

```
{3694.5, Null}
```

In[14]:=

```
PrintOptimal[alg3]
```

There are 8 optimal families of 3-dimensional Lie subalgebras.

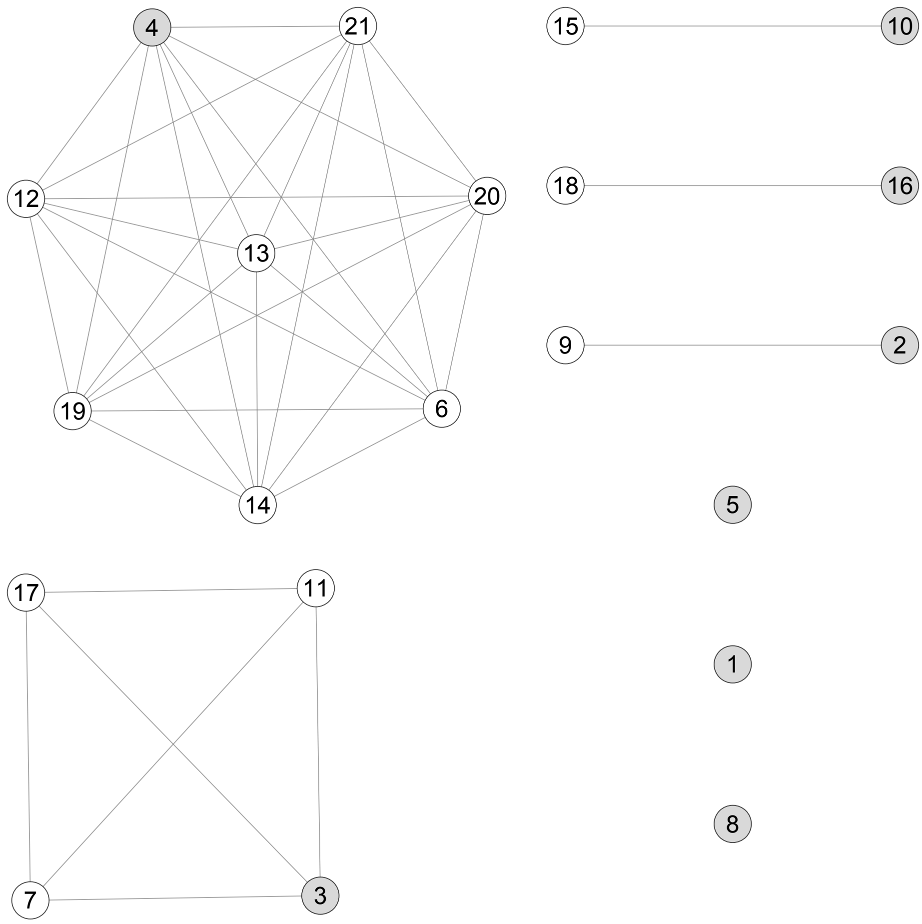
Out[14]=

$\{\{\Xi_1, \Xi_2, \Xi_3\}, \{\Xi_1, \Xi_2, \Xi_4\}, \{\Xi_1, \Xi_3, \Xi_4\}, \{\Xi_2, \Xi_3, \Xi_4\},$
$\{\Xi_2, \Xi_4, \Xi_5\}, \{\Xi_3, \Xi_5, \Xi_6\}, \{\Xi_1, \Xi_2, \Xi_3 + a_1 \Xi_4\}, \{\Xi_1 + \alpha_1 \Xi_5, \Xi_2, \Xi_4\}\}$

In[15]:= `PrintGraph[alg3, 1]`

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3\},\ 2 \to \{\Xi_1, \Xi_2, \Xi_4\},\ 3 \to \{\Xi_1, \Xi_3, \Xi_4\},\ 4 \to \{\Xi_2, \Xi_3, \Xi_4\},$
$5 \to \{\Xi_2, \Xi_4, \Xi_5\},\ 6 \to \{\Xi_3, \Xi_4, \Xi_5\},\ 7 \to \{\Xi_3, \Xi_4, \Xi_6\},\ 8 \to \{\Xi_3, \Xi_5, \Xi_6\},\ 9 \to \{\Xi_4, \Xi_5, \Xi_6\},$
$10 \to \{\Xi_1, \Xi_2, \Xi_3 + a_1 \Xi_4\},\ 11 \to \{\Xi_1, \Xi_2 + \alpha_1 \Xi_3, \Xi_4\},\ 12 \to \{\Xi_2, \Xi_3 + \alpha_1 \Xi_5, \Xi_4\},$
$13 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_2, \Xi_4\},\ 14 \to \{\Xi_2 + \alpha_1 \Xi_3, \Xi_4, \Xi_5\},\ 15 \to \{\Xi_3 + a_1 \Xi_4, \Xi_5, \Xi_6\},$
$16 \to \{\Xi_1 + \alpha_1 \Xi_5, \Xi_2, \Xi_4\},\ 17 \to \{\Xi_3 + \alpha_1 \Xi_5, \Xi_4, \Xi_6\},\ 18 \to \{\Xi_2 + \alpha_1 \Xi_6, \Xi_4, \Xi_5\},$
$19 \to \{\Xi_3 + \alpha_1 \Xi_6, \Xi_4, \Xi_5\},\ 20 \to \{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_5, \Xi_2, \Xi_4\},\ 21 \to \{\Xi_2 + \alpha_1 \Xi_3 + \alpha_2 \Xi_6, \Xi_4, \Xi_5\}\}$

Out[15]=



In[16]:= `Timing[alg4 = SubAlgebra[cs, {{}, {}}, 4];]`

```
There are 12 4-D families of subalgebras to be analyzed.
Done.
```

Out[16]=

`{10 242.2, Null}`

In[17]:= `PrintOptimal[alg4]`

```
There are 5 optimal families of 4-dimensional Lie subalgebras.
```

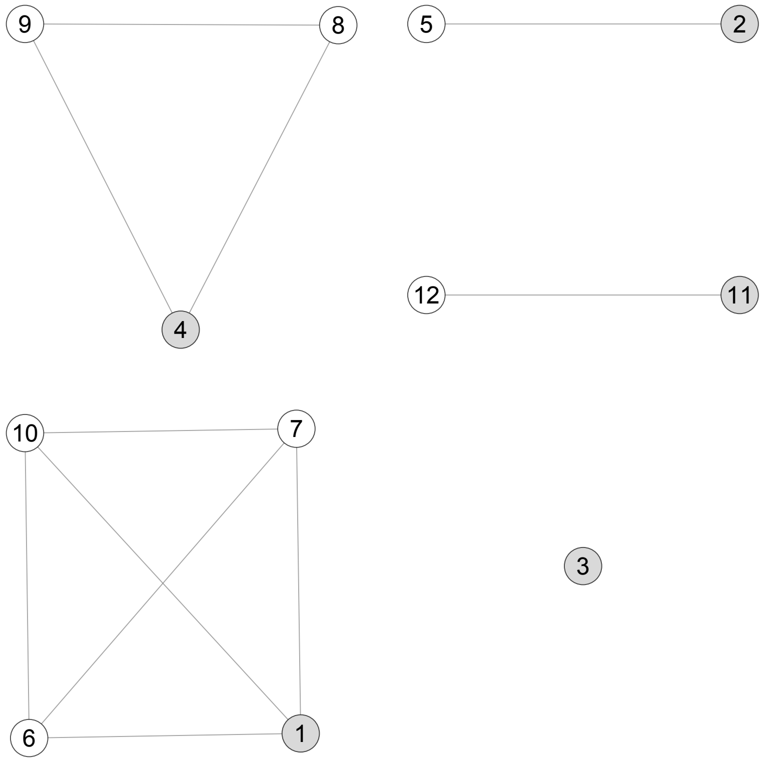
Out[17]=

$\{\{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}, \{\Xi_1, \Xi_2, \Xi_4, \Xi_5\},$
$\{\Xi_1, \Xi_3, \Xi_4, \Xi_6\}, \{\Xi_2, \Xi_3, \Xi_4, \Xi_5\}, \{\Xi_1 + \alpha_1 \Xi_6, \Xi_2, \Xi_4, \Xi_5\}\}$

In[18]:= **PrintGraph[alg4, 1]**

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}, 2 \to \{\Xi_1, \Xi_2, \Xi_4, \Xi_5\}, 3 \to \{\Xi_1, \Xi_3, \Xi_4, \Xi_6\}, 4 \to \{\Xi_2, \Xi_3, \Xi_4, \Xi_5\},$
$5 \to \{\Xi_2, \Xi_4, \Xi_5, \Xi_6\}, 6 \to \{\Xi_3, \Xi_4, \Xi_5, \Xi_6\}, 7 \to \{\Xi_1, \Xi_2, \Xi_3 + \alpha_1 \Xi_5, \Xi_4\},$
$8 \to \{\Xi_2, \Xi_3 + \alpha_1 \Xi_6, \Xi_4, \Xi_5\}, 9 \to \{\Xi_1 + \alpha_1 \Xi_3, \Xi_2, \Xi_4, \Xi_5\}, 10 \to \{\Xi_2 + \alpha_1 \Xi_3, \Xi_4, \Xi_5, \Xi_6\},$
$11 \to \{\Xi_1 + \alpha_1 \Xi_6, \Xi_2, \Xi_4, \Xi_5\}, 12 \to \{\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_6, \Xi_2, \Xi_4, \Xi_5\}\}$

Out[18]=



In[19]:= **Timing[alg5 = SubAlgebra[cs, {{}, {}}, 5];]**

```
There are 2 5-D families of subalgebras to be analyzed.
Done.
```

Out[19]=

{4217.06, Null}

In[20]:= **PrintOptimal[alg5]**

```
There are 1 optimal families of 5-dimensional Lie subalgebras.
```

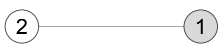
Out[20]=

$\{\{\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5\}\}$

In[21]:= **PrintGraph[alg5, 1]**

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5\}, 2 \to \{\Xi_2, \Xi_3, \Xi_4, \Xi_5, \Xi_6\}\}$

Out[21]=



In[22]:= **alg = {alg1, alg2, alg3, alg4, alg5}**

Out[22]=

```
{{{{1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

```
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0,
 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
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 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0,
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 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0,
 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
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 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
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   {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0},
   {0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0},
   {1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0}, {1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1}},
  {{1, 2, 4, 8}, {1, 2, 8, 16}, {1, 4, 8, 32}, {2, 4, 8, 16},
   {2, 8, 16, 32}, {4, 8, 16, 32}, {1, 2, 20, 8}, {2, 36, 8, 16},
   {5, 2, 8, 16}, {6, 8, 16, 32}, {33, 2, 8, 16}, {37, 2, 8, 16}},
  {{{0, 0, 2, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 2, 0}, {0, 0, 1, 0, 0, 0}, {0, -1, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}},
```

```
{{0, 0, 0, 0, 0, 4}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {-4, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, -2}, {0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {2, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 2}, {0, 0, 0, 0, 1, 0},
 {0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0}, {0, -2, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 2},
 {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, -2, 0, 0, 0}}}},
{{{1, 1}, {1, 1}}, {{1, 2, 4, 8, 16}, {2, 4, 8, 16, 32}},
 {{{0, 0, 2, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 2, 0}, {0, 0, 1, 0, 0, 0}, {0, -1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 4}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {-4, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, -2}, {0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {2, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 2}, {0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0}, {0, -2, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 2},
  {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, -2, 0, 0, 0}}}}}
```

In[23]:= **SessionTime[]**

Out[23]=

20 553.186403