Algebra of Lie symmetries of viscous Burgers' equation

In[1]:=
```
SetDirectory[NotebookDirectory[]];
```

In[2]:=
```
<< "SymboLie.wl"
```

```
SymboLie (v. 1.6) - A Package for determining Optimal Systems of Lie Subalgebras.
```

In[3]:=
```
gens = {{1, 0, 0}, {0, 1, 0}, {0, t, 1}, {2 t, x, -u}, {t^2, t x, x - t u}};
vars = {t, x, u};
pars = {{}, {}};
```

In[4]:=
```
cs = StructureConstants[gens, vars];
```

In[5]:=
```
CommutatorTable[cs] // MatrixForm
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \Xi_2 & 2\,\Xi_1 & \Xi_4 \\ 0 & 0 & 0 & \Xi_2 & \Xi_3 \\ -\Xi_2 & 0 & 0 & -\Xi_3 & 0 \\ -2\,\Xi_1 & -\Xi_2 & \Xi_3 & 0 & 2\,\Xi_5 \\ -\Xi_4 & -\Xi_3 & 0 & -2\,\Xi_5 & 0 \end{pmatrix}$$

In[6]:=
```
Timing[alg1 = SubAlgebra[cs, pars, 1];]
```

```
There are 31 1-D families of subalgebras to be analyzed.

Done.
```

Out[6]= {24.8638, Null}

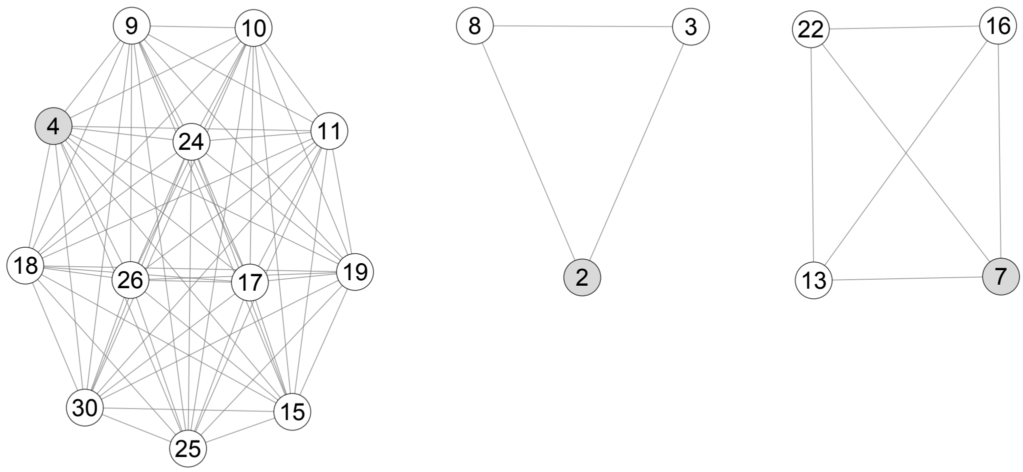
In[7]:=
```
PrintOptimal[alg1]
```

```
There are 5 optimal families of 1-dimensional Lie subalgebras.
```

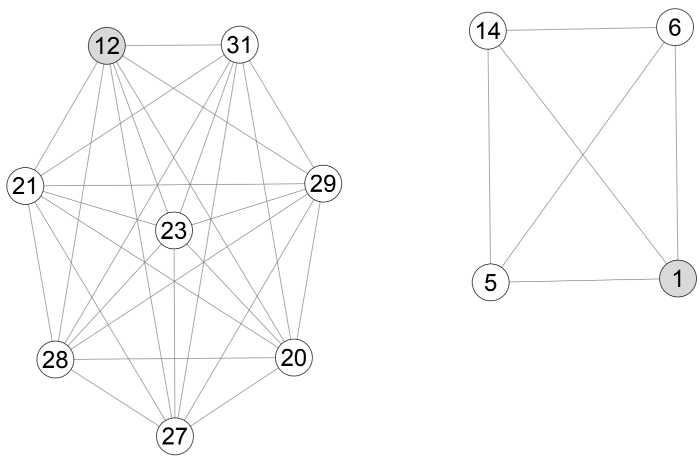
Out[7]= $\{\{\Xi_1\}, \{\Xi_2\}, \{\Xi_4\}, \{\Xi_1 + \alpha_1\,\Xi_3\}, \{\Xi_1 + \alpha_1\,\Xi_5\}\}$

In[8]:= **PrintGraph[alg1, 1]**

$\{1 \to \{\Xi_1\},\ 2 \to \{\Xi_2\},\ 3 \to \{\Xi_3\},\ 4 \to \{\Xi_4\},\ 5 \to \{\Xi_5\},\ 6 \to \{\Xi_1 + \alpha_1\,\Xi_2\},\ 7 \to \{\Xi_1 + \alpha_1\,\Xi_3\},$
$8 \to \{\Xi_2 + \alpha_1\,\Xi_3\},\ 9 \to \{\Xi_1 + \alpha_1\,\Xi_4\},\ 10 \to \{\Xi_2 + \alpha_1\,\Xi_4\},\ 11 \to \{\Xi_3 + \alpha_1\,\Xi_4\},\ 12 \to \{\Xi_1 + \alpha_1\,\Xi_5\},$
$13 \to \{\Xi_2 + \alpha_1\,\Xi_5\},\ 14 \to \{\Xi_3 + \alpha_1\,\Xi_5\},\ 15 \to \{\Xi_4 + \alpha_1\,\Xi_5\},\ 16 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_3\},$
$17 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4\},\ 18 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_4\},\ 19 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4\},\ 20 \to \{\Xi_1 + \alpha_1\,\Xi_2 + a_1\,\Xi_5\},$
$21 \to \{\Xi_1 + \alpha_1\,\Xi_3 + a_1\,\Xi_5\},\ 22 \to \{\Xi_2 + \alpha_1\,\Xi_3 + a_1\,\Xi_5\},\ 23 \to \{\Xi_1 + \alpha_1\,\Xi_4 + \alpha_2\,\Xi_5\},$
$24 \to \{\Xi_2 + \alpha_1\,\Xi_4 + a_1\,\Xi_5\},\ 25 \to \{\Xi_3 + \alpha_1\,\Xi_4 + \alpha_2\,\Xi_5\},\ 26 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + a_1\,\Xi_4\},$
$27 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + a_1\,\Xi_5\},\ 28 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_4 + a_1\,\Xi_5\},\ 29 \to \{\Xi_1 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + a_1\,\Xi_5\},$
$30 \to \{\Xi_2 + \alpha_1\,\Xi_3 + \alpha_2\,\Xi_4 + \alpha_3\,\Xi_5\},\ 31 \to \{\Xi_1 + \alpha_1\,\Xi_2 + \alpha_2\,\Xi_3 + \alpha_3\,\Xi_4 + a_1\,\Xi_5\}\}$



Out[8]=



In[9]:= **Timing[alg2 = SubAlgebra[cs, pars, 2];]**

```
There are 17 2-D families of subalgebras to be analyzed.
Done.
```

Out[9]= {128.3, Null}

In[10]:= **PrintOptimal[alg2]**

```
There are 5 optimal families of 2-dimensional Lie subalgebras.
```

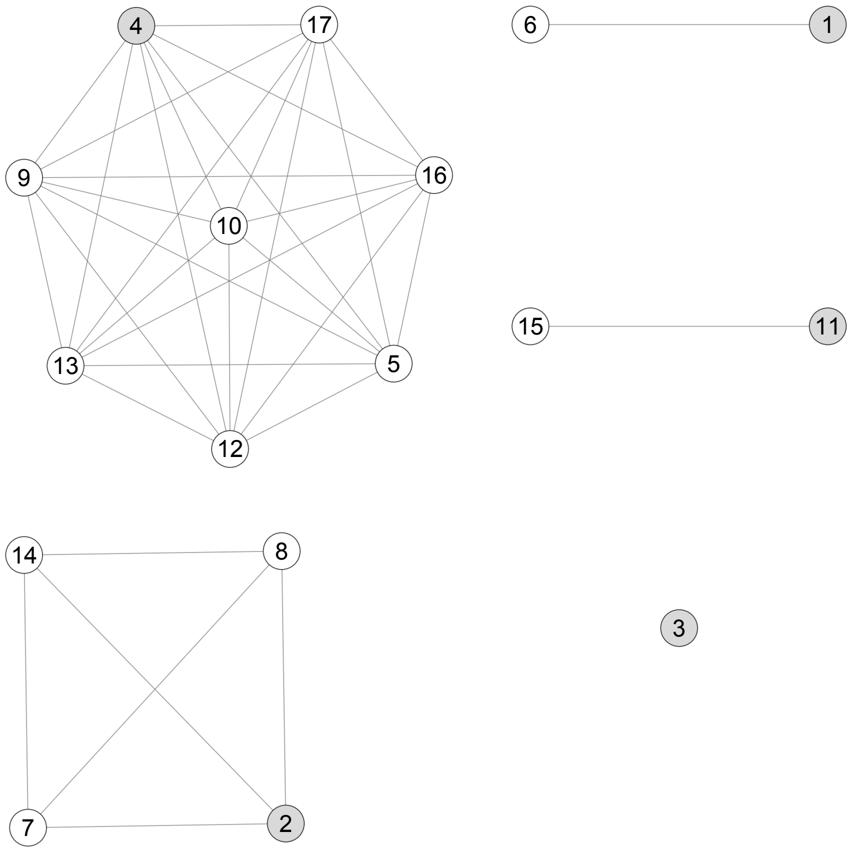
Out[10]=

$\{\{\Xi_1, \Xi_2\}, \{\Xi_1, \Xi_4\}, \{\Xi_2, \Xi_3\}, \{\Xi_2, \Xi_4\}, \{\Xi_1 + \alpha_1\,\Xi_3, \Xi_2\}\}$

In[11]:= **PrintGraph[alg2, 1]**

{1 → {Ξ_1, Ξ_2}, 2 → {Ξ_1, Ξ_4}, 3 → {Ξ_2, Ξ_3}, 4 → {Ξ_2, Ξ_4}, 5 → {Ξ_3, Ξ_4}, 6 → {Ξ_3, Ξ_5}, 7 → {Ξ_4, Ξ_5}, 8 → {Ξ_1, $\Xi_2 + \alpha_1 \Xi_4$}, 9 → {Ξ_2, $\Xi_3 + \alpha_1 \Xi_4$}, 10 → {Ξ_3, $\Xi_4 + \alpha_1 \Xi_5$}, 11 → {$\Xi_1 + \alpha_1 \Xi_3$, Ξ_2}, 12 → {$\Xi_1 + \alpha_1 \Xi_4$, Ξ_2}, 13 → {$\Xi_2 + \alpha_1 \Xi_4$, Ξ_3}, 14 → {$\Xi_3 + \alpha_1 \Xi_4$, Ξ_5}, 15 → {$\Xi_2 + \alpha_1 \Xi_5$, Ξ_3}, 16 → {$\Xi_1 + \alpha_1 \Xi_3 + \alpha_2 \Xi_4$, Ξ_2}, 17 → {$\Xi_2 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5$, Ξ_3}}

Out[11]=



In[12]:= **Timing[alg3 = SubAlgebra[cs, pars, 3];]**

There are 12 3-D families of subalgebras to be analyzed.

Done.

Out[12]=

{959.884, Null}

In[13]:= **PrintOptimal[alg3]**

There are 5 optimal families of 3-dimensional Lie subalgebras.

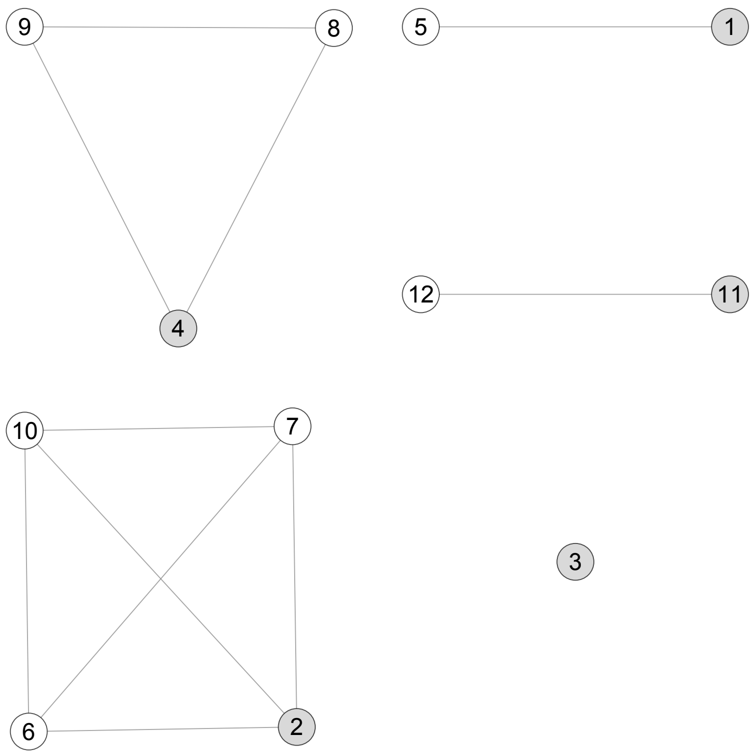
Out[13]=

{{Ξ_1, Ξ_2, Ξ_3}, {Ξ_1, Ξ_2, Ξ_4}, {Ξ_1, Ξ_4, Ξ_5}, {Ξ_2, Ξ_3, Ξ_4}, {$\Xi_1 + \alpha_1 \Xi_5$, Ξ_2, Ξ_3}}

In[14]:=
```
PrintGraph[alg3, 1]
```

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3\}, 2 \to \{\Xi_1, \Xi_2, \Xi_4\}, 3 \to \{\Xi_1, \Xi_4, \Xi_5\}, 4 \to \{\Xi_2, \Xi_3, \Xi_4\}, 5 \to \{\Xi_2, \Xi_3, \Xi_5\}, 6 \to \{\Xi_3, \Xi_4, \Xi_5\}, 7 \to \{\Xi_1, \Xi_2, \Xi_3 + \alpha_1 \Xi_4\}, 8 \to \{\Xi_2, \Xi_3, \Xi_4 + \alpha_1 \Xi_5\}, 9 \to \{\Xi_1 + \alpha_1 \Xi_4, \Xi_2, \Xi_3\}, 10 \to \{\Xi_2 + \alpha_1 \Xi_4, \Xi_3, \Xi_5\}, 11 \to \{\Xi_1 + \alpha_1 \Xi_5, \Xi_2, \Xi_3\}, 12 \to \{\Xi_1 + \alpha_1 \Xi_4 + \alpha_2 \Xi_5, \Xi_2, \Xi_3\}\}$

Out[14]=



In[15]:=
```
Timing[alg4 = SubAlgebra[cs, pars, 4];]
```

There are 2 4-D families of subalgebras to be analyzed.

Done.

Out[15]=

{120.888, Null}

In[16]:=
```
PrintOptimal[alg4]
```

There are 1 optimal families of 4-dimensional Lie subalgebras.

Out[16]=

$\{\{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}\}$

In[17]:=
```
PrintGraph[alg4, 1]
```

$\{1 \to \{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}, 2 \to \{\Xi_2, \Xi_3, \Xi_4, \Xi_5\}\}$

Out[17]=

In[18]:=
```
alg = {alg1, alg2, alg3, alg4}
```

Out[18]=

{{{{1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 1, 0, 0, 0, 0, 1,

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0,
 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0}, {1, 0, 0, 0, 1, 1, 0, 0,
 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1,
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 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1}},
{{1}, {2}, {4}, {8}, {16}, {3}, {5}, {6}, {9}, {10}, {12}, {17},
 {18}, {20}, {24}, {7}, {11}, {13}, {14}, {19}, {21}, {22},
 {25}, {26}, {28}, {15}, {23}, {27}, {29}, {30}, {31}},
{{{0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {-1, 0, 0, 0, 0},
  {0, -1, 0, 0, 0}, {0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 1},
  {0, 0, 0, -1, 0}, {0, 0, 1, 0, 0}, {0, -1, 0, 0, 0}},
```

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  {{0, 0, 0, 0, 1}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
   {0, 0, 0, 0, 2}, {0, 0, 0, -2, 0}}}},
{{{1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
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  {0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1},
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  {4, 24}, {5, 2}, {9, 2}, {10, 4}, {12, 16}, {18, 4}, {13, 2}, {26, 4}},
 {{{0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {-1, 0, 0, 0, 0}, {0, -1, 0, 0, 0},
   {0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, -1, 0},
   {0, 0, 1, 0, 0}, {0, -1, 0, 0, 0}}, {{0, 0, 0, 0, 1}, {0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 2}, {0, 0, 0, -2, 0}}}},
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 {{1, 2, 4}, {1, 2, 8}, {1, 8, 16}, {2, 4, 8}, {2, 4, 16}, {4, 8, 16},
  {1, 2, 12}, {2, 4, 24}, {9, 2, 4}, {10, 4, 16}, {17, 2, 4}, {25, 2, 4}},
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  {{0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {-1, 0, 0, 0, 0}, {0, -1, 0, 0, 0},
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   {0, 0, 1, 0, 0}, {0, -1, 0, 0, 0}}, {{0, 0, 0, 0, 1}, {0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 2}, {0, 0, 0, -2, 0}}}},
{{{1, 1}, {1, 1}}, {{1, 2, 4, 8}, {2, 4, 8, 16}},
 {{{0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {-1, 0, 0, 0, 0}, {0, -1, 0, 0, 0},
   {0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, -1, 0},
   {0, 0, 1, 0, 0}, {0, -1, 0, 0, 0}}, {{0, 0, 0, 0, 1}, {0, 0, 0, 0, 0},
```

```
{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0},
{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 2}, {0, 0, 0, -2, 0}}}}}
```

In[19]:=
```
SessionTime[]
```

Out[19]=
```
1190.357148
```