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A two immiscible liquids penetration model for surface-driven capillary flows

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This is a mathematical and numerical study of liquid dynamics in a horizontal capillary. We present a two-liquids model which takes into account the effects of real phenomena like the outside flow dynamics. Moreover, we report on results obtained by an adaptive numerical method.

1 The two liquids penetration model

The model considered here is of interest for the non-destructive control named "liquid penetrant testing" used in the production of airplane parts as well as in many industrial applications where the detection of open defects is of interest.

We have derived, within the one-dimensional approximation and for a cylindrical capillary section, a two immiscible liquids penetration model for surface-driven capillary flows. With reference to Fig. 1, we consider a column of liquid 1, usually water, of fixed length ℓ_0 entrapped within a horizontal cylindrical capillary of radius R and length L. At the left end of the capillary we have a reservoir filled with a penetrant liquid 2 and the moving interface between the two liquids is denoted by ℓ.

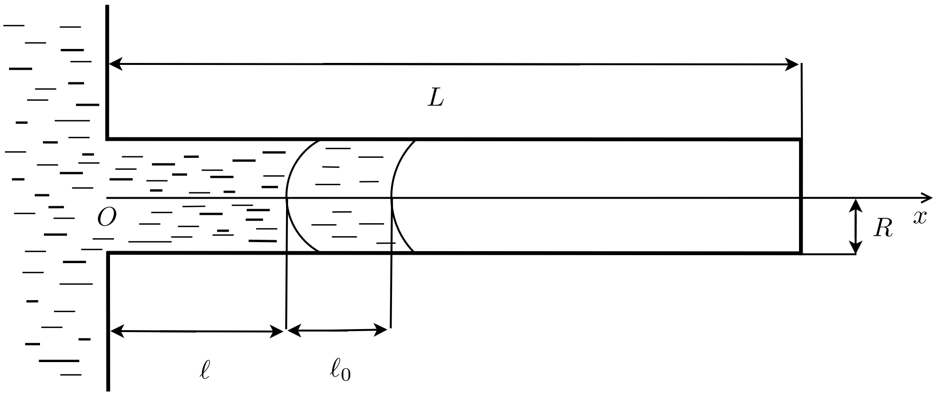

Fig. 1 Draft of a cylindrical capillary section.

The governing equation is given by the second order ordinary differential equation

$$[\rho_1 \ell_0 + \rho_2(\ell + cR)] \frac{d^2\ell}{dt^2} + \rho_2 \left(\frac{d\ell}{dt}\right)^2 = 2\frac{\gamma_1 \cos\vartheta_1 + \gamma_{12}\cos\vartheta_{12}}{R} - 8\frac{\mu_1\ell_0 + \mu_2\ell}{R^2}\frac{d\ell}{dt} , \tag{1}$$

where ρ_1 and ρ_2 are the densities, μ_1 and μ_2 are the dynamic viscosities of the two liquids, γ_1 and γ_{12} are the surface free energy for the liquid 1-air and the liquid 1-liquid 2 interfaces, ϑ_1 and ϑ_{12} are corresponding menisci contact angles, and t is time. We have taken into account here the coefficient of apparent mass $c = O(1)$, introduced by Szekely et al. [1].

Moreover, in the case of a vertical capillary, the gravity action should be considered, by adding to the left hand side of equation (1) the term

$$\pm(\rho_1\ell_0 + \rho_2\ell)g , \tag{2}$$

where the plus or mines sign have to be used when the liquid reservoir is over or below the cavity, respectively.

Full details on the derivation of this model, as well as extensive numerical simulations, will be reported elsewhere. For an overview on capillary dynamics the interested reader is referred to the recent book by de Gennet et al. [2] and the references quoted therein.

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2 Numerical simulations

We report on the numerical results for the model (1) in the case of an infinite capillary, with the natural initial conditions $\ell(0) = 0$, $\frac{d\ell}{dt}(0) = 0$, see Kornev and Neimark [3], and parameters listed in Table 1. Several test cases were considered where

Liquid	Viscosity μ (mPa-s)	Density ρ (Kg/m^3)	Surface tension γ (mN/m)	Contact angle ϑ
Silicon oil	500	980	21.1	0°
Ethanol	1.17	780	21.6	0°
Ether	0.3	710	16.6	0°
Mixture	0.77	955	57	10°
Water	1	998	71.8	0°

Table 1 Liquids parameters according to the recent survey by Kornev and Neimark [3].

water was always in front to the other liquids. For the sake of brevity we report only the numerical results for two of the mentioned test cases. With reference to Fig. 2, the left frame shows the case with ethanol and water, and the right frame the one with mixture and water. We notice that the monotone functions are $\ell(t)$ and $\ell_0(t)$; the fast transient of the first derivative

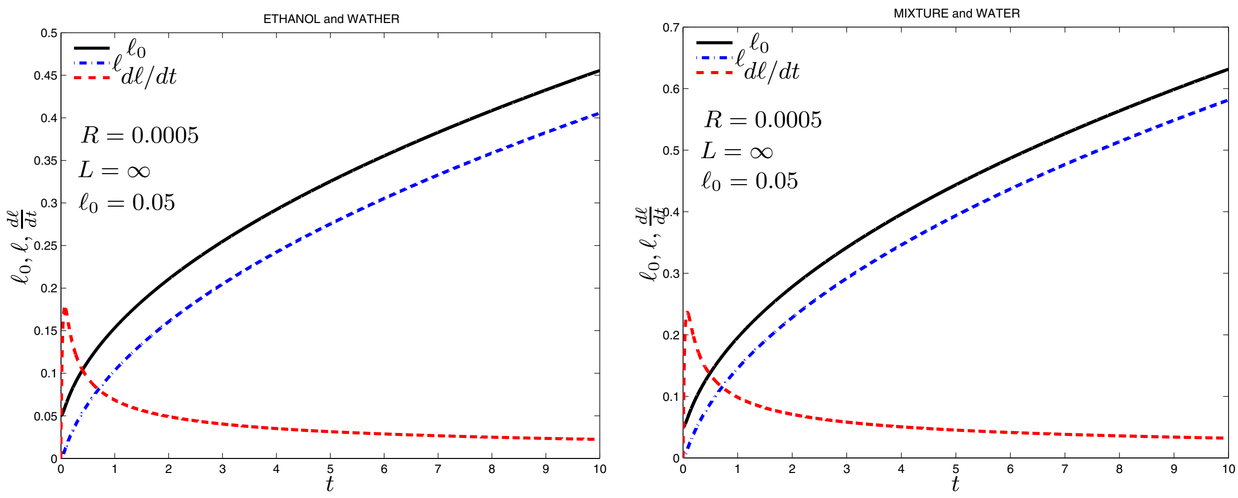

Fig. 2 Open capillary with $\ell_0 = 0.05$, and $R = 0.0005$: on the left ethanol and water; on the right mixture and water.

of ℓ is characteristic for this kind of problems. The comparison between the two frames clearly explains how the mixture, having a higher surface tension and a lower viscosity with respect to the ethanol, reaches a dipper distance inside the capillary.

These results were obtained by a fourth order Runge-Kutta's method implemented with an adaptive procedure developed by Jannelli and Fazio [4].

Finally, we notice that a preliminary academic numerical test for the reduced model valid in the case of $\ell_0 = 0$ (and $\gamma_{12} = 0$) is reported in the note by Cavaccini et al. [5].

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