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Mathematical Aspects in Non Equilibrium Systems from Micro To Macro

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Book of Abstracts

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Contents

Probabilistic quantization and its potential applications outside Physics

Luigi Accardi
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(joint work with Yun Gang Lu)

The purpose of this talk is to formulate the main theses advocated by the probabilistic quantization program, so that everyone can judge for themselves whether the more technical developments of this program really support these theses. For 100 years the term quantization has been related to physics. Now we understand that quantization is a universal phenomenon in classical probability.

Physics has shown that the quantum formalism is very powerful in dealing with physical phenomena. Now we have to learn how to harness this power in favour of every eld in which probability plays a role. This practically includes every human activity.

The term quantum technology has been understood up to now as: applications of quantum theory to technology. But now we understand that, from the mathematical point of view, what in physics is called quantization should be called: gaussian quantization in the boson case; Bernoulli quantization in the fermion case. But these are very special cases of probabilistic quantization.

And now we know that every measure with all moments canonically determines its own quantization rules. So nowadays the term quantum technology should be understood as: applications of probabilistic quantization to technology (but see a caveat at the end of this abstract).

There is a nontrivial feedback for physics.

There are strong mathematical results proving that:

In dimensions ≥ 2, in a truly interacting quantum theory Heisenberg commutation relations must be replaced by the intrinsic probabilistic commutation relations associated with a given measure (see again the caveat at the end of this abstract).

The basic new idea is that nonlinearity is not only coded into the interaction potentials, but also into the commutation relations. A caveat to the above program comes from the fact that there are many examples, coming from the stochastic limit of quantum theory, which show that not all commutation relations that encode nonlinear interactions arise from probabilistic quantization.

So, probabilistic quantization is not the end of the story (but who believes that, in science, there will ever be an end of the story ?)

This is a delicate issue, but there will certainly not be time to discuss it in my talk. For reasons of time I will discuss only the case of a single Rvalued random variable with all moments, but the theory works for every classical random eld with all moments (so also quantum eld theory is covered).

If time allows, I will try to explain why there is a phase transition in diculty from dimension 1 to dimensions ≥ 2, why this phase transition has not been noticed up to now in the literature, and which are the new qualitative fea tures (with respect to standard quantization) that emerge in this transition.

A mathematical model for the EPR phenomenon

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We consider a couple of quantum non-identical particles that are constrained to one spatial dimension and are initially entangled in the position and momentum variables. They do not interact with each other while one of them, i.e. particle 1, interacts with a spin locate at a fixed position. We introduce a scaling that involves the strength and the width of the interaction, the energy of the spin, and the Planck's constant. We show that, at the first order in the small scaling parameter, the spin flip is correlated with the motion of particle 2, that does not interact with it. This is a version of the effect theoretically foreseen by Einstein, Podoslki and Rosen in their famous paper of 1935. The research project is in collaboration with L. Barletti (Florence) and A. Teta (Rome).

The Hopfield model: Looking Backward, Looking Forward

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The Hopfield network is a paradigmatic model for both biological neural networks (where it mimics associative memory) and artificial neural networks (where it is used for retrieval tasks). Since its introduction in the 1980s, it has been extensively studied by mathematicians and physicists, however, a fundamental question—the existence of the thermodynamic limit—has remained elusive.
In the first part of this talk, I will outline a strategy to tackle this long-standing problem. In the second part, I will introduce a new class of tasks that this type of network can address: pattern disentanglement.

Single-particle diffraction with a hydrodynamic pilot-wave model

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Abstract

Millimetric liquid droplets can self-propel by virtue of the wave field that they generate by bouncing on a vibrating liquid surface [1]. These walking droplets exhibit several quantum-like behaviors [2]. A seminal experiment suggested that single-particle diffraction may be obtained when a droplet crosses apertures between submerged barriers [3], a behavior questioned by subsequent experiments [4, 5, 6]. We used a hydrodynamic pilot-wave model [7] to explore the interaction of walking droplets with linear barriers. The statistical distributions of the droplets are wavelike and generally exhibit multiple peaks depending on the obstacle geometry. We propose a mechanism for this wavelike behavior of such classical particles and provide a detailed comparison with diffraction in optics and quantum mechanics.

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Magnetically Confined quantum systems – promising candidates for studying Coulomb entanglement

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The Wigner theory provides a seamless transition from quantum to classical electron transport thus allowing to model from fully coherent phenomena, such as interference, to scattering dominated processes, such as Brownian motion. It has been successfully applied to single-electron control, quantum transport and quantum information processing.

The standard theory is formulated by the Weyl transform in terms of the Wigner potential, defined by the electric potential V related to the electric field by $\mathbf{E} = -\nabla V$. When the electric field $\mathbf{E}(x) = E_0(x - x_0)\hat{x}$ is linear, the Wigner potential reduces to the electrical force which governs the Newtonian trajectories - the characteristics of the Liouville operator in the Wigner equation. Accordingly, in control problems the confining potential applied to manipulate the quantum state can be chosen to be quadratic which results in a force term, that acts on the mean position and allows to manipulate the variance [1]. In particular a quantum harmonic oscillator (QHO)[2], having a ground state given by a minimum-uncertainty Gaussian function centered in x_0 of width $\sigma_x = \sqrt{\hbar/(2m\omega_0)}$ where $\omega_0 = \sqrt{eE_0/m}$, is stationary. However if σ_x assumes a different value the quantum state is no more stationary: σ_x becomes time-dependent and oscillates with a frequency ω_0. Magnetic fields $\mathbf{B}(\mathbf{r})$ can be involved by the the Weyl-Stratonovich transform which provides a gauge invariant Wigner equation, formulated entirely in terms of the electromagnetic fields [3]. For uniform magnetic fields $\mathbf{B} = B_0\hat{z}$, the equation reduces to the Boltzmann equation driven by the Lorenz force $(e/m)\cdot(\mathbf{P}\times\mathbf{B})$. This allows conveniently to compare the quantum confinement obtained from uniform magnetic fields and with the confinement obtained by a 2D isotropic linear electric field, $\mathbf{E}(\mathbf{r}) = E_0(x - x_0)\hat{x} + E_0(y - y_0)\hat{y}$. The analysis shows that in the case of uniform magnetic fields the ground state is degenerate [4], but one of the degenerate state is again a minimum-uncertainty Gaussian state with a variance, $\sigma_x = \sigma_x = \sigma = \sqrt{\hbar/(m\omega_c)}$, where $\omega_c = eB_0/m$ is the cyclotron frequency. One of the differences with the electric case is that if we choose a different value of σ for the initial state the evolution of $\sigma(t)$ has a frequency $\omega_c/2$, a half of the electric case if we choose $\omega_c = \omega_0$. Another important difference with the electric case is that we can place a Gaussian ground state in any point of the plane $(x, y) \in \mathbb{R}^2$; in the electric case, on the contrary, the ground state is only one and is centered in the point (x_0, y_0) where the field is zero. The opportunity of placing more than one confined quantum state ($\mathbf{P} = 0$ and σ constant or oscillating) allows us to consider this system ideal to investigate the effects of Coulomb interaction between the states, which give rise to entanglement. (Coulomb entanglers).

The results obtained with quantum particle simulations [5] manifest the above notions. Their stability makes the numerical approach a feasible candidate for analysis of entanglement.

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A Class of Exactly Solvable Real and Complex PT Symmetric Reflectionless Potentials

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Abstract

We consider the question of the number of exactly solvable complex but PT-invariant reflectionless potentials with N bound states. By carefully considering the X_m rationally extended reflectionless potentials, we argue that the total number of exactly solvable complex PT-invariant reflectionless potentials are $2[(2N-1)m+N]$.

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Super-activation of back-flow of information in collisional models

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Even if a single non-Markovian quantum dynamics does not exhibit back-flow of information (BFI) from the environment to an open quantum system, its tensor product with itself does. Such super-activation of BFI (SBFI) at the level of two open quantum systems is certainly a quantum phenomenon, though not necessarily due to entanglement. Its emergence and features will be studied by means of a collisional model where the environment ancillas form a classical Markov chain. In particular, SBFI will be specifically related to the system-environment correlations.

Quantum Observables and their Mathematical Representation

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Abstract

The operational analysis of the measurement process shows that positive operator valued measures (POVMs) are the appropriate mathematical tool to describe the statistics of the outcomes of the measurement of a quantum observable. In this framework, self-adjoint operators (or spectral measures) represent a subclass of the class of quantum observables. The resulting generalization opens the way to a rigorous analysis of questions like the approximate joint measurement of incompatible observables, the unavoidable disturbance due to the measurement process, the definition of a time observable, the description of the photon localization, the analysis of non-repeatable measurements and the description of open systems, etc.

From the mathematical viewpoint, POVMs were introduced by Naimark in his attempt to analyze the problem of the self-adjoint extension of non-self-adjoint (but symmetric) operators. They are defined a s o perator v alued m easures which are σ-additive in the weak operator topology, the range being made of self-adjoint positive operators less than the identity. In particular, POVMs provide an integral representation of symmetric operators.

The aim of the talk is to review the mathematical properties and the physical meaning of positive operator valued measures (POVMs) and to elaborate on some recent results. We start with a general introduction to the topic. Then we analyze the measurement process in order to motivate the introduction of POVMs in quantum mechanics and show that they provide a generalization of the standard definition of quantum observable as a self-adjoint operator. We then focus on fuzzy observables which are described by commutative POVMs and are relevant to quantum mechanics because they model certain standard forms of noise in quantum measurements and provide optimal approximators as marginals in joint measurements of incompatible observables (e.g., Position and Momentum). Indeed, one of the advantages of POVMs with respect to self-adjoint operators is that two self-adjoint operators are compatible if and only if they commute whereas two non-commuting POVMs can be compatible. This is at the basis of a rigorous mathematical description of the approximate joint measurement of incompatible observables.

On the mathematical side, we first r eview t he r ole o f P OVMs a s i ntegral representation for symmetric (but non self-djoint) operators. We then focus on the fuzzification-defuzzification pr ocess th at ch aracterizes th e co mmutative POVMs. Next, we show that two well known integral representations of commutative POVMs are equivalent. Finally, time permitting, we analyze the case of POVMs that are covariant with respect to a symmetry group. For such POVMs a Radon-Nikodym derivative with respect to a scalar measure can be introduced providing a representation of quantum mechanics on phase space.

Theory of swarming as extended criticality

L. L. Bonilla, A. Carpio, R. González-Albaladejo*
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Collective biological systems display power laws for macroscopic quantities and are fertile probing grounds for statistical physics. Besides power laws, natural insect swarms present strong scale-free correlations, suggesting closeness to phase transitions. Swarms exhibit imperfect dynamic scaling: their dynamical correlation functions collapse into single curves when written as functions of the scaled time tξ^{-z} (ξ : correlation length, z: dynamic exponent), but only for short times. Triggered by markers, natural swarms are not invariant under space translations. Measured static and dynamic critical exponents differ from those of equilibrium and many nonequilibrium phase transitions. Here we consider the discrete-time Vicsek model with particles (insects) confined by a harmonic potential and calibrated by data from experiments.The model exhibits time periodic, quasiperiodic and chaotic attractors and interesting scale-free lines among chaotic attractors as the number of insects N increases [1].

In the parameter space of confinement strength and alignment noise, lines separating chaotic single cluster and multicluster swarms and lines separating chaotic from non chaotic attractors are scale-free and coalesce at the same rate as N>>1 to the line of zero confinement.

They characterize a new and previously unsuspected scale-free-chaos phase transition involving chaotic attractors. For finite N, these lines belong to a novel extended critical region and finite size scaling arguments allow to calculate critical exponents of power laws for correlation length and time and for susceptibility [1,2]. These power laws imply that the unmeasurable confinement strength is proportional to the insect perception range measured in natural swarms [2]. Observations of natural swarms occur at different times and under different atmospheric conditions, which we mimic by considering mixtures of data on different critical lines and N [2]. Unlike results of other theoretical approaches, our numerical simulations reproduce the previously described features of natural swarms and yield static and dynamic critical exponents that agree with observations.

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Application of Quantum-Markov Open System Models to Human Cognition and Decision

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Markov processes, such as random walk models, have been successfully used by cognitive and neural scientists to model human choice behavior and decision time for over 50 years. Recently, quantum walk models have been introduced as an alternative way to model the dynamics of human choice and confidence across time. Empirical evidence points to the need for both types of processes, and open system models provide a way to incorporate them both into a single process. However, some of the constraints required by open system models present challenges for achieving this goal.The purpose of this article is to address these challenges and formulate open system models that have good potential to make important advancements in cognitive science.

Quantum MEP hydrodynamical model for charge and phonons transport

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A well known procedure to get quantum hydrodynamical models for charge transport is to resort to the Wigner equations and deduce the hierarchy of the moment equations as in the semiclassical approach. If one truncates the moment hierarchy to a finite order, the resulting set of balance equations requires some closure assumption because the number of unknowns exceed the number of equations. In the classical and semiclassical kinetic theory a sound approach to get the desired closure relations is that based on the Maximum Entropy Principle (MEP).

Here I am going to present a quantum hydrodynamical model which is valid for a general energy band (case not present in literature) considering a closure of the moment system deduced by the Wigner equation resorting to a quantum version of MEP. Explicit formulas for quantum correction at order $\hbar^2$ are obtained with the aid of the Moyal calculus for silicon and graphene removing the limitation that the quantum corrections are based on the equilibrium Wigner function as in the cases already discussed in literature. As an application, quantum correction to the mobilities are deduced.

Using the same strategy, an energy transport model for phonons and an explicit form of the thermal conductivity with quantum correction up to $\hbar^2$ order are obtained.

Quantum hybrids

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We investigate the existence, nonexistence, and properties of normalized ground states of a nonlinear Schrödinger equation on a hybrid manifold, consisting of simple manifolds but with different spatial dimensionality. The nonlinearity is focusing and subcritical. The energy comprises nonlinear Schrödinger energies with contact interactions on both the manifolds, coupled by an additional quadratic term. Ground states are minimizers of the energy at a fixed mass. We identify a key criterion for existence: ground states form if confinement near the junction is energetically favorable compared to escaping to infinity. We demonstrate that ground states exist for small and large mass values, contingent on the strength of interactions and coupling at the origin. Nonexistence arises under strong repulsive interactions on both components and weak coupling. Additionally, we explore the qualitative properties of the ground states. In cases where the system consists of two connected planes, we show that ground states are either concentrated on one plane or distributed across both, with mass distribution and singularity strength depending on the matching condition parameters and nonlinear powers.

joint work with R. Adami, L. Tentarelli, F. Boni, I. di Giorgio

Second law and the possibility of spatial interactions in higher-grade materials

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Mathematical Aspects in Non Equilibrium Systems: from Micro To Macro

Erice, October 30th - November 4th, 2024

Abstract

We investigate the thermodynamic compatibility of weakly nonlocal materials, with constitutive equations depending on the third spatial gradient of the deformation. Generalized constitutive laws for the stress tensor and the specific entropy are obtained. For onedimensional elastic solids in the presence of small deformations, the possibility of propagation of thermomechanical disturbances is explored. It is proved that the relaxation of the heat flux does not ensure the hyperbolicity of the system of balance laws. For the equilibrium problem, an explicit form of the displacement is calculated. Comparison is made with the equilibrium theory of Korteweg fluids.

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Mathematical Aspects in Non Equilibrium Systems From Micro to Macro

Analysis of thermoelectric effects at nanoscale: the Onsager's method and the Liu's technique in comparison

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The analysis of coupled transport phenomena is one of the most outstanding aspects of non-equilibrium thermodynamics. In this talk the attention is put on thermoelectricity, i.e., the coupling of heat and electricity. We propose a theoretical model which goes beyond the usual relations employed at macro-scale to describe thermoelectric effects. It introduces both non-local and non-linear effects which should be taken into account in view of the possible applications of thermoelectric effects at nanoscale.

From a theoretical point of view, in the particular case of the proposed model it is shown the equivalence between two classical techniques for the exploitation of the second law of thermodynamics, i.e., the Onsager's method and the Liu's technique. The proposed model is employed to investigate how non-local and non-linear effects may influence the propagation of waves.

Dynamically decoupling of Markovian quantum channels

Paolo Facchi
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We show that every finite-dimensional quantum system with Markovian (i.e. GKLS-generated) time evolution has an autonomous unitary dilation which can be dynamically decoupled. This highlights the role of dilations in the control of quantum noise. We construct our dilation via a time-dependent version of Stinespring in combination with Howland’s clock Hamiltonian and certain point-localised states, which may be regarded as a C*-algebraic analogue of improper bra-ket position eigenstates.

Phase transitions in many-body open systems

Rosario Fazio
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Thanks to recent impressing experimental progress, the investigation of non-equilibrium properties of driven-dissipative quantum systems has received an impressive boost. Rydberg atoms in optical lattices, systems of trapped ions, exciton-polariton condensates, cold atoms in cavities, arrays of coupled QED cavities, are at present the most intensively investigated experimental platforms to this aim. While in many cases the effect of an external bath is detrimental for quantum information protocols, it has been shown that by proper engineering of the environment, quantum manipulations are possible. I will address features of the steady-state phase diagram displaying a variety of critical phenomena peculiar of non-equilibrium systems.

Energy cutoffs, noncommutativity, fuzzyness: the case of the $O(D)$-covariant fuzzy spheres

Gaetano Fiore
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Abstract

Projecting a quantum theory onto the Hilbert subspace of states with energies below a cutoff $\overline{E}$ generally leads to an effective theory with modified observables, including a noncommutative space(time). Adding a confining potential well V makes the corresponding Hilbert space finite-dimensional. If V has a very sharp minimum on a submanifold N of the original space(time) M, and we carefully choose $\overline{E}$, we get a quantum theory on the lower dimensional N; this can be seen as implementing a *quantum constraint*. I will in particular report on our application of this procedure to spheres $S^d \subset \mathbb{R}^D$ of radius $r = 1$ $(D = d{+}1 > 1)$: making $\overline{E}$ and the depth of the well depend on (and diverge with) $\Lambda \in \mathbb{N}$ we obtain fuzzy spheres S^d_Λ covariant under the *full* orthogonal groups $O(D)$; the commutators of the coordinates depend only on the angular momentum, as in Snyder noncommutative spaces. As $\Lambda \to \infty$ the Hilbert space dimension diverges, $S^d_\Lambda \to S^d$, and we recover ordinary quantum mechanics on S^d. Focusing on $d = 1, 2$, we also discuss uncertainty relations, localization of states, diagonalization of the space coordinates and construction of coherent states. These models might be suggestive for effective models in quantum field theory, quantum gravity or condensed matter physics.

Mathematical aspects in non equilibrium systems: from micro to macro

Organizers

Fabio Bagarello, Francesco Oliveri, Vittorio Romano

Title of Presentation

Quantum ideas in micro level decision making and macro level financial system analysis.

Presenter

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Abstract

Non equilibrium systems, like financial markets, can be approached from both a micro and a macro level.

On the micro level of financial decision making, an agent needs to act in an environment with multitudes of dynamically changing pieces of information. In a very simple case, a Bayesian decision agent, who is keen to make a certain investment, is confronted with two pieces of new information that both individually reduce the agent's inclination to make that investment. In such a situation, it would be natural to assume that the simultaneous consideration of both pieces of information should further reduce the agent's initial enthusiasm. Surprisingly, general statements like this about the double conditional probability require substantial additional assumptions. We investigate schemes of assumptions in classical Bayesian probability and quantum probability that allow `reasonable` conclusions about the double conditional probability.

On the macro level traditional quantitative methods typically consist of multivariate linear regression models that offer limited insight into causal mechanisms. Since the seminal work of John Bell, causal models are used in quantum foundations to understand the constraints for hidden variable models that aim at reproducing quantum statistics. Notably, such causal approach allows to address counterfactual questions. It can be fully formalized and then used as a test to rule out competing causal models of price formation on the macro level.

Modeling Epidemics with Memory Effects Using the Open Quantum System Approach

Francesco Gargano*

We present an operatorial framework to model epidemic dynamics using the formalism of open quantum systems. Specifically, we adopt the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation for the density operator, including memory effects to describe the time evolution of the system. We assume that each population -infected, recovered, and deceased-is modeled as a fermionic mode. Memory effects are incorporated to account for the time delays between infection, recovery, and death, reflecting the realistic non-instantaneous nature of these processes. The Lindblad operators introduced govern the irreversible transitions between these states, while the Hamiltonian is assumed to be null due to the lack of reciprocal interactions. The resulting system of equations reduce to the evolutions of the populations of the density operator. We show with numerical experiments that the positivity of the evolution for the density operator is preserved.

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Quantum circuit complexity for linearly polarised light

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I present a form of quantum circuit complexity that extends to open systems. To illustrate the methodology, I focus on a basic model where the projective Hilbert space of states is depicted by the set of orientations in the Euclidean plane. Specifically, I investigate the dynamics of mixed quantum states as they undergo interactions with a sequence of gates. This approach involves the analysis of sequences of real 2X2 density matrices. Such a mathematical model is physically exemplified by the Stokes density matrices, which delineate the linear polarisation of a quasi-monochromatic light beam, and the gates, which are viewed as quantum polarisers, whose states are also real 2X2 density matrices. The interaction between polariser-linearly polarised light is construed à la Von Neumann within the context of this quantum formalism. Each density matrix for the light evolves in a way analogous to a Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) process during the time interval between consecutive gates. Notably, when considering an upper limit for accuracy, it is shown that the optimal number of gates follows a power-law relationship.

From a submitted article co-authored by E. Curado A. Maioli, D. Noguera (CBPF, Rio), S. Faci (UFF, Rio), T. Koide (UFRJ, Rio).

A fermionic operatorial model for a newtork of agents with competitive and cooperative interactions

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An operatorial model of a system consisting in a network of N agents interacting each other with mechanisms that can be thought of as cooperative and/or competitive is presented [1]. We associate to each agent an annihilation, creation and number fermionic operator, and interpret the mean values of the number operators over an initial condition as measures of the agents' wealth status. The dynamics is ruled by a suitable quadratic Hamiltonian operator with the addition of specific rules (depending on the variations of the mean values of the observables) that periodically modify some of the involved parameters and able to adjust periodically the model to the agents behavior, i.e., we move in the framework of the so called (H,ρ)-induced dynamics approach [2]. The nature and the strength of the interactions change according to the model evolution, as well as the so called "inertia parameters" associated to the agents. The cooperative interactions are clearly responsible to avoid the insorgence of strong inequalities in the network .

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Asymptotic State Convertibility in Semigroup-Majorization based Quantum Resource Theories

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In several quantum resource theories, state conversions are governed by a majorization relation between probability vectors. After discovering this connection for the resource theory of entanglement, Nielsen conjectured that as system dimensionality increases, the fraction of convertible pairs of states vanishes. This conjecture was analytically proven only recently [1], following systematic numerical investigations [2], for the resource theories of both entanglement [1] and coherence [3]. However, in both cases, it remains unclear how quickly the fraction of convertible pairs decays. The conjecture can be extended to the resource theory of thermal operations, where state conversions are governed by a generalized majorization relation. In this case, we prove [4] the conjecture in the limit of low temperature, and provide an explicit expression for the fraction of convertible pairs, valid at any dimension. Furthermore, we consider situations where exact conversion is not achievable, and compute explicitly the distribution of the maximal success probability between random initial and target states.

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Quantum transport on Bethe lattices with non-Hermitian sources and a drain

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We consider quantum transport on a tight-binding model on the Bethe lattice of a finite generation, or the Cayley tree, which may model the energy transport in a light-harvesting molecule. As a new feature to analyze the quantum transport, we add complex potentials for sources on the peripheral sites and for a drain on the central site. We find that the eigenstates that can penetrate from the peripheral sites to the central site are quite limited to the number of generation. All the other eigenstates are localized around the peripheral sites and cannot reach the central site. The former eigenstates can carry the current, which reduces the problem to the quantum transport on a parity-time ($\mathcal{PT}$)-symmetric tight-binding chain. When the number of links is common to all generations, the current takes the maximum value at the exceptional point where two eigenstates coalesce to a zero-energy state, which emerges because of the non-Hermiticity due to the $\mathcal{PT}$-symmetric complex potentials. As we introduce randomness in the number of links in each generation of the tree, the resulting linear chain is a random-hopping tight-binding model. We find that the current reaches its maximum not exactly but approximately for a zero-energy state, although it is no longer located at an exceptional point in general.

Ubiquitous Quantum: from genetics and biological evolution to cognition, psychology, decision making, and social science

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This is introduction to quantum-like modeling, applications of the methodology and formalism of quantum theory outside of physics, in cognition, psychology, decision making, social and political sciences, economics and finance, genetics and evolutionary biology. It is important to point out that systems under consideration are macroscopic. So, quantum-like theory should be sharply distinguished from It starts with the brief introduction to quantum theory (so one need not be educated in this field); the information and probabilistic counterparts will be highlighted. The motivations for quantum-like modeling will be presented and illustrated by applications to agents' irrational behavior - disjunction and order effects. The latter in combination with another psychological effect, the response replicability effect, leads to the use of theory of quantum instruments. Quantum-like approach is used in biology, e.g., in genetics and epigenetics, for modeling adaptive dynamical interactions with environment, e.g., phenotype's generation. Recently such models found applications in medical diagnostics of neurological diseases.

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Measuring contextuality in investment preferences

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Abstract

Contextuality in preference formation can have many facets. Behavioural finance studies focus on contextuality that steams from presentation format or framing, other studies evaluate the impact of order effects, joint and separate evaluation of random variables, such as stock prices and portfolio versus single stock assessment of returns. In this work, we aim to experimentally measure the degree of contextuality in a set of paired choices over stock investments.

We follow the general framework of Dzhafarov & Kujala (2013) that allows us to detect contextuality and its degree in a system of random variables. The main premise of this framework is that new context changes the identity of the variable and hence, transforms it into another random variable, even when probability distribution did not change.

Our findings cater to the existing studies in behavioural finance and economics, whereby we measure the existence of contextuality in choices over real stocks, part of the Dow Jones Index and some countries' stock indexes. We also run a regression model to assess the impact of beliefs about the future performance of stocks, self-assessed risk attitude and other factors.

The Role of Positive Vectors in Indefinite Inner Product Spaces

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The aim of this talk is to emphasize the significant role played by positive vectors in an indefinite inner product space, $\mathcal{H}$. Specifically, we demonstrate that the properties of a linear operator W can be uniquely determined by its restriction to the nonlinear set of positive vectors in $\mathcal{H}$. Our research was partly inspired by a remark in [1] during the study of manifestly non-self-adjoint $\mathcal{PT}$-symmetric Hamiltonians: 'One can try to formulate a quantum theory associated with $\mathcal{PT}$-symmetric Hamiltonians by insisting that physical states must have positive norm. This leads to a quantum mechanics defined on a nonlinear state space. Such investigations are of interest, but the existence of negative-norm eigenstates still leaves open serious interpretational issues.

The results presented in this talk are the outcome of the joint work with Fabio Bagarello.

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Statistically validated comorbidity networks

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We analyze a comprehensive unbiased local database of medical records of a large number of patients treated at the Hospital District of southwest Finland for any reason between January 2004 and July 2019 . We apply an age-cohort and sex-cohort approach to a total of 628 831 patients, construct the diagnostic history of each patient, and include the information how old the patient was when diagnosed for the first time with each diagnosis coded according to International Classification of Diseases. Starting from the bipartite dataset of patient-ICD code, we construct for each cohort a projected comorbidity network of those ICD codes that that simultaneously present in at least one patient. These cohort specific projected networks are relatively dense and performing community detection and a dismantling strategy relatively inefficacy. We therefore highlight on top of these networks the statistically validated networks composed by links rejecting the null hypothesis of random connection of diseases maintaining the degree of disease heterogeneity observed in real data. Community detection is effective in statistically validated networks and dismantling procedure more effective than in the underlying projected network. Our results highlight ICD (i) specificity of different sex-age cohorts and (ii) specificity of the dismantling procedure of the statistically validated networks providing a potential indications of efficacy for policy makers.

Optimal charge distribution for the Vlasov-Poisson system by an external electric field

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Confining and manipulating plasmas are of great importance in various applications, such as plasma cooling, satellite electric propulsion, and energy production in fusion reactors. We introduce a novel optimization framework aimed at designing equilibrium configurations for a Vlasov-Poisson system subjected to an external electric control field. The framework involves introducing an ensemble objective functional with Tikhonov regularization, utilizing a differential constraint represented by a nonlinear elliptic equation modeling equilibrium solutions, conducting a complete theoretical analysis of the optimization problem, and validating the proposed procedure numerically.

The Vlasov-Poisson system governs the transport of a particle distribution function under a self-consistent force, modeled by a Poisson equation representing electrostatic forces between charged particles. The system serves as an alternative to the Boltzmann description for gases of charged particles with long-range Coulomb interaction. The focus is on steady equilibria solutions characterized by the nonlinear elliptic Poisson-Boltzmann equation, including an external electric control field, to concentrate particle density in selected regions.

The formulation involves a constrained optimization problem, with the differential constraint being the nonlinear Poisson-Boltzmann equation. An ensemble objective functional is chosen to reflect the statistical nature of the Vlasov-Poisson system. Tikhonov regularization is employed to ensure well-posedness of the optimization problem and appropriate regularity of the optimal control field.

We provides theoretical investigations into the optimization problem, proving existence of optimal control fields and discussing first-order optimality conditions. We also outline a numerical implementation strategy involving the approximation of the Poisson-Boltzmann equation and its optimization adjoint, as well as assembling the H_1 gradient for a gradient-based iterative solution procedure.

Finally, we present numerical experiments, demonstrating the framework's ability to manipulate steady equilibrium plasma configurations using an external electric field.

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Optimal control of quantum dynamics in phase space

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Controlling the quantum state of an isolated or interacting correlated atoms emerges as a promising research activity in modern quantum information science. Two-particle entanglement constitutes an essential resource for quantum technologies such as quantum teleportation, quantum cryptography, and quantum metrology. Ensembles of ultracold neutral atoms have proven to be a powerful tool for producing entanglement in atoms. They can be well isolated from the environment, enabling long-lived manipulation of coherent states. Wigner function formalism provides an ideal tool to describe quantum evolution of systems with continuous degrees of freedom and has been widely used to represent entangled states in quantum optics. In this contribution, I discuss some recent results concerning the application of the Wigner formalism to optimal control of atoms moving in the presence of external fields. I will discuss the formulation of the optimal control problem in terms of the Wigner formalism, the well posedness of the nonlinear set of equations, and the numerical discretization scheme.
My results show the possibility to induce strong correlations in trapped interacting atoms. The robustness of the control is investigated by considering the presence of an external source of noise weakly interacting with the atoms.

Analysis of nonlinear one dimensional Guyer-Krumhansl heat equation

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As is commonly known that Fourier's law has limits, so it is inevitable to find the correct extension and determine the possibilities of practical implementations. The Guyer-Krumhansl model is promising and could be the standard model in future engineering practice, but it would not be possible without a thorough investigation and understanding of its mathematical properties and the study of nonlinear effects resulting from non-constant material parameters. For this reason, we investigate the effects of non-constant material parameters, such as thermal conductivity and relaxation time. In fact, given the practical importance of this approach, it is essential to discover and understand the effects to study real problems.

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Wigner–Boltzmann Monte Carlo simulation of thermionic cooling devices based on resonant-tunneling AlGaAs/GaAs heterostructure

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Temperature stabilization by means of electronic refrigerators has attracted much interest, as a scalable, reliable and green option. Research in this field has mainly focused on two approaches, which rely either on thermoelectricity (based on the Peltier effect) or thermionic emission of hot carriers. Thermionic devices were proposed as an alternative to thermoelectric devices, able to reduce the effects of carrier scattering and lattice thermal conduction.

We study by means of full quantum simulations the operating principle and performance of a semiconductor heterostructure refrigerator combining resonant tunneling filtering and thermionic emission.

Our model takes into account the coupling between the electric and thermal currents by self-consistently solving the Wigner-Boltzmann transport equation using the signed Monte Carlo method and the heat equation. We show that the device can achieve relatively high cooling power values, while in the considered implementation, the maximum lattice temperature drop is severely limited by the thermal conductivity of the constituting materials. Quantum simulations demonstrate that cooling properties of such tilted barrier devices are significantly improved in the out-of-equilibrium regime, where the thermionic cooling concept offers its best efficiency.

Optimal control of a semiclassical Boltzmann equation for charge transport in graphene

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The construction of semiconductor nanostructures requires efficient and accurate methodologies that accommodate physical and engineering details as well as the required functionality of the resulting device. Within this framework, we developed a model for controlling charge transport in graphene for the design of new devices. The semiclassical Boltzmann equation represents an accurate model for graphene simulation that accommodates microscopic effects. In this model, a natural control mechanism is an external electric field due to an applied voltage at the metallic contacts. With this control, we can manipulate the momentum and current through the graphene. In our work, these tasks are formulated with ensemble cost functionals. This choice appears natural since ensemble cost functionals can be interpreted as ensemble averages of potential functions determining the desired mean trajectory and final target. We focus on a space-homogeneous time-dependent electric control field whose cost in the cost functional is given by a quadratic H1 term that guarantees continuous control functions. Well-posedness of the control problem is proved, and the characterization of optimal controls by an optimality system is discussed. The latter is approximated by a discontinuous Galerkin scheme and the optimization is performed by using a nonlinear conjugate gradient method. Results of numerical experiments are presented that demonstrate the ability of the proposed control framework to design control fields.

EXCITATION ENERGY TRANSFER THROUGH A SPIN-CHAIN

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Biological processes reveal abilities which are very impressive and cannot be adequately explained with the only use of traditional (classical) approaches. A certain amount of quantum mechanics is thought to be used by nature in order to enhance the efficiency of the underlying processes. The question which arises then is: *How can quantum features survive in an open quantum system subject to a permanent environmental noise*? In this talk I shall present a mathematical model for the illustration of the excitation energy transfer in photosynthesis complexes, with the aim to study the "constructive" interplay between the coherent quantum excitation transfer and the vibrational classical motion of the underlying molecular structure. The model is based on the Schrödinger equation associated to a time-dependent Hamiltonian, describing the propagation of an absorbed photon through a spin-chain towards the reaction center of the photosynthesis apparatus (Spin-Boson model). Let me also mention that efficient transfer of (quantum) excitation/energy is essential not only in this field of quantum biology but also in various other fields of application, ranging from molecular technology, quantum computer to quantum heat-engines. Thus the presented model is rather general.

Quantum Decay at Short, Intermediate, and Long Times

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The time evolution of an unstable quantum system is quadratic at short times (the so-called quantum Zeno region), and follows a power law at long times. The quantum Zeno region has been confirmed in a plethora of experiments, with very different quantum systems and experimental platforms. On the other hand, the power law regime is elusive and very difficult to observe, essentially because the initial states are strongly depleted by intermediate exponential evolution.

After analyzing the problem from a theoretical point of view, we will review some experiments and discuss concepts and applications, focusing on analogies with statistical mechanics.

"Our freedom, characteristic of the very idea of experiment": Complementarity and decision in quantum and quantum-like theories

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Bohr's concept of complementarity is commonly associated with the mutual exclusivity of establishing or predicting certain phenomena or events, such as those of the exact position and the momentum measurements (mutually exclusive by the uncertainty relation). This is, however, only part of Bohr's concept, *as defined by him*, which has two other equally important parts: (1) the possibility of defining each phenomenon or event separately at any given point; and (2) the necessity of considering all of them at different moments of time in accounting for the totality of phenomena in quantum physics. This more complex structure of the concept also explains why "the wave-particle complementarity," commonly used as an example, does not properly correspond to Bohr's concept and was not invoked by Bohr.

The focus of this paper is (2), to be considered as defined by our decision to perform either one or the other experiment, defining the corresponding complementary events, by using one or the other of the two possible but mutually incompatible experimental arrangements. Of course, as noted by Bohr, "our freedom of handling measuring instruments … characteriz[es] the very idea of experiment" in all physics. This paper replaces the concept of "freedom" with a more general concept of "decision," which allows for a freedom in making a decision but may also limit this freedom. Indeed, in contrast to classical physics or relativity, in quantum or quantum-like theories (theories that use the mathematics of quantum theory in fields other than physics), this freedom is both essential to complementarity and is limited by it insofar as *deciding* to perform one complementarity experiment, enabling the corresponding predictions, irrevocable precludes simultaneously performing the other and making the predictions that it would enable. We are no longer free to do so. Complementarity changes the epistemological nature and (in the sense defined in this paper) topological structure of decision and, correlatively, causality, in quantum and quantum-like theories vis-à-vis classical physics or relativity, which do not contain complementarity or, correlatively, the uncertainty relations. Thus, one in principle can, is *free*, to decide to measure both the position and the momentum of an object considered at any given point in time. This paper will discuss this transformation, with an additional emphasis on the quantum-mechanical mathematical formalism, rarely discussed in considering complementarity. Crucial to this situation are, the paper shows, the fundamental relationships between classical (macro) systems and quantum systems (which can be both micro and macro). This argument also allows the paper to address the relationships between probability (including Bayesian probability) and statistics in quantum and quantum-like theories, and correlativity, between subjective and objective aspects of decision there, additionally grounding them in quantum information theory.

H-theorems for dense inert and reactive mixtures with application to global in time existence of solutions

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In this talk I show existence of an H-function and global in time existence of solutions for the SRS model. Simple reacting spheres (SRS) has been developed by N. Xystris and J. S. Dahler (J. Chem. Phys. **68** (1978), 387–401) and further advanced by J. S. Dahler and L. Quin in (J. Chem. Phys. (1995) **103**, 725–750 and **118** (2003), 8396–8404). Recently, J. Polewczak and A. J. Soares considered modified SRS model that also preserves conservation of angular momentum in reactive collisions (Kinetic and Related Models (KRM), **10** (2017), No. 2, pp. 513 - 539).

In the model, the molecules behave as if they were single mass points with two internal states of excitation. Collisions may alter the internal states. Reactive and non-reactive collision events are considered to be hard spheres-like. A four component mixture A, B, A^*, B^* is considered; the chemical reactions are of the type

$$A + B \rightleftharpoons A^* + B^*.$$

Here, A^* and B^* are distinct species from A and B. I use the indices 1, 2, 3, and 4 for the particles A, B, A^*, and B^* respectively. Furthermore, m_i and d_i denote the mass and the diameter of the i-th particle, $i = 1,\dots,4$, and reactions take place when the reactive particles are separated by a distance $\sigma_{12} = \frac{1}{2}(d_1 + d_2) = \sigma_{34} = \frac{1}{2}(d_3 + d_4)$ and the kinetic energy associated with the relative motion exceeds the activation energy: $(1/2)\mu_{is}\big(\langle\epsilon, v - w\rangle\big)^2 \geq \gamma_i$, with $\mu_{is} = \frac{m_i m_s}{m_i + m_s}$, the reduced masses. Here the activation energies γ_i have the properties $\gamma_2 = \gamma_1$ and $\gamma_3 = \gamma_4 = E_{abs} - \gamma_1$ and the absorbed energy $E_{abs} = E_3 + E_4 - E_1 - E_2 > 0$, where $E_i > 0$, $i = 1, \dots 4$, is the energy of i-th particle associated with its internal degrees of freedom. The conservation of mass has the form

$$m_1 + m_2 = m_3 + m_4 = M,$$

where m_i denotes the mass of the i-th particle, $i = 1,\dots,4$. For $i = 1,2,3,4$, $f_i(t,x,v)$ denotes the one-particle distribution function of the ith component of the reactive mixture. The SRS kinetic system has the form

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} = E_i + R_i, \quad i = 1,2,3,4,$$

where E_i is the non-reactive (hard-sphere) and R_i is the reactive collision operator:

$$E_i = \sum_{s=1}^{4}\left\{\sigma_{is}^2 \iint\limits_{\mathbb{R}^3\times\mathbb{S}^2}\left[f_{is}^{(2)}(t,x,v',x-\sigma_{is}\epsilon,w') - f_{is}^{(2)}(t,x,v,x+\sigma_{is}\epsilon,w)\right]\langle\epsilon, v-w\rangle\, d\epsilon dw\right\} \tag{1}$$

$$- \beta\sigma_{ij}^2 \iint\limits_{\mathbb{R}^3\times\mathbb{S}^2}\left[f_{ij}^{(2)}(t,x,v',x-\sigma_{ij}\epsilon,w') - f_{ij}^{(2)}(t,x,v,x+\sigma_{ij}\epsilon,w)\right]\Theta(\langle\epsilon,v-w\rangle - \Gamma_{ij})\langle\epsilon,v-w\rangle d\epsilon dw,$$

$$R_i = \beta\sigma_{ij}^2 \iint\limits_{\mathbb{R}^3\times\mathbb{S}^2}\left[\left(\frac{\mu_{ij}}{\mu_{kl}}\right)^2 f_{kl}^{(2)}(t,x,v_{ij}^{\odot},x-\sigma_{ij}\epsilon,w_{ij}^{\odot}) - f_{ij}^{(2)}(t,x,v,x+\sigma_{ij}\epsilon,w)\right]\Theta(\langle\epsilon,v-w\rangle - \Gamma_{ij})\langle\epsilon,v-w\rangle d\epsilon dw,$$

with $\mathbb{S}^2 = \{\epsilon \in \mathbb{R}^3 : |\epsilon| = 1, \langle\epsilon, v-w\rangle \geq 0\}$, and $f_{is}^{(2)} = g_{is}\, f_i\, f_s$ approximates the density of pairs of particles in collisional configurations. Here $g_{ij}(r_1, r_2)$ is the mixtures' pair correlation function for the system of hard spheres in ***non–uniform equilibrium***. $g_{ij}^{(2)}$ depends on local densities $n_i(t,x) = \int f_i(t,x,v)\,dv$.

The second term in (1), with β in front of it singles out those pre-collisional states that are energetic enough to result in the reaction, and thus preventing double counting of the events in the collisional integrals.

One can also look at $0 \leq \beta \leq 1$ as a probability for energetic enough particles to undergo reactive collisions.

In the case $\beta = 0$, E_i, in (1), is the inert collisional operator for hard-sphere mixture.

Here, $\Gamma_{ij} = \sqrt{2\gamma_i/\mu_{ij}}$, and Θ is the Heaviside step function. The pairs of post-reactive velocities, $(v_{ij}^{\odot}, w_{ij}^{\odot}) = (v^{\ddagger}, w^{\ddagger})$ for $i,j = 1,2$, and $(v_{ij}^{\odot}, w_{ij}^{\odot}) = (v^{\dagger}, w^{\dagger})$ for $i,j = 3,4$. Pairs of indices (i,j) and (k,l) are from the set of quadruples (i,j,k,l):

$$\{(1,2,3,4), (2,1,4,3), (3,4,1,2), (4,3,2,1)\}.$$

Additionally, when $\beta = 0$ (no reactive terms), system (1) becomes an inert dense mixture of hard spheres (in this case one can consider any number of species with arbitrary masses) and this work also provides the first global in time existence of a solution for the corresponding system.

The quantum sequential sampler

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Fallacies in decision making, such as the conjunction fallacy, have been driven the development of cognitive theory for several decades. Yet we are now at a point such that the state-of-the-art models, including the original Busemeyer et al. (2011) one, are becoming indistinguishable. Are there empirical data which help push further the debate and challenge existing theory? We present results from the, as of today, most extensive dataset, showing evidence for new fallacies, which present difficulties from all current models. We propose a new quantum model, incorporating a sequential sampling mechanism, which provides good fit against our results and compares favourably to the
leading Bayesian model.

Equilibrium and out-of-equilibrium dynamics in long-range Hamiltonian systems

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In this paper I will review the main features of equilibrium and out-of-equilibrium dynamics in long-range Hamiltonian systems. In particular the dynamics of the Hamiltonian Mean Field (HMF) model will be addressed in connection to chaoticity and Tsallis nonextensive statistical mechanics [1-8].

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Analysis of nonlinear generalizations of heat equation in heat pulse propagation

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The nonlinear terms resulting from temperature dependence of thermal conductivity and relaxation time are not negligible when studying applications to nanosystems.

Two nonlinear generalizations of the Maxwell-Cattaneo-Vernotte equation for thermal transport with relaxation effects are introduced.

A comparison of the consequences of these different nonlinear Cattaneo type equations on thermal pulse propagation is obtained and discussed. The differences in the corresponding velocities and in the corresponding heights of the perturbation peaks turn out to be significant and could be experimentally detected.

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Modeling and Analysis of semiconductor laser diodes

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This presentation focuses on the mathematical modeling of semiconductor lasers using coupled drift-diffusion models integrated with the Helmholtz equation. In particular, we will discuss the assumptions needed to prove the existence and uniqueness results for this kind of model. Moreover, we will describe the role of optical power as determined by the photon balance equation and its sensitivity to semiconductor potentials.

Resonances crossing effect and quantum sensor of electric fields

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While the phenomenon of the exact crossing of energy levels is a rarely occurring event, in the case of quantum resonances associated with metastable states this phenomenon is much more frequent and various scenarios can occur. When there is an exact crossing of the imaginary parts of the resonances in a two-level quantum system subject to an external DC electric field, then a damped beating phenomenon occurs, which is absent if there is no such crossing. This fact, tested numerically on an explicit one-dimensional model, suggests the possibility of designing quantum sensors to determine in a very simple way whether the external field strength has an assigned value or not.

Coherent States of Graphene Layer with and without a $\mathcal{PT}$-symmetric Chemical Potential

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Abstract

In this paper we construct different classes of coherent and bicoherent states for the graphene tight-binding model in presence of a magnetic field, and for a deformed version where we include a $\mathcal{PT}$-symmetric chemical potential V. In particular, the problems caused by the absence of a suitable ground state for the system is taken into account in the construction of these states, for $V = 0$ and for $V \neq 0$. We introduce ladder operators which work well in our context, and we show, in particular, that there exists a choice of these operators which produce a factorization of the Hamiltonian. The role of broken and unbroken $\mathcal{PT}$-symmetry is discussed, in connection with the strength of V.

Gauge principle and gauge invariance issues in truncated Hilbert spaces: Application to cavity QED

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The interaction between quantized electromagnetic fields in cavities and natural or artificial atoms has played a crucial role in developing our understanding of light-matter interactions and second generation quantum technologies [1].

In the last ten years, new regimes for light-matter coupling of cavity-QED have been explored in several settings, wherein the light-matter coupling rate becomes comparable to (ultrastrong coupling) or even exceeds (deep-strong coupling) the resonance frequencies of the light or matter components [2]. These coupling regimes can give rise to new physical effects and applications, and they challenge our understanding of cavity QED. Fundamental issues like the proper definition of subsystems, their quantum measurements, the structure of light-matter ground states, and the analysis of time-dependent interactions are subject to gauge ambiguities that lead to even qualitatively distinct predictions [3,4]. The resolution of these ambiguities is important for understanding and designing next-generation quantum devices that can operate in extreme coupling regimes [5].

In particular it has been shown that usual approximations as the truncation of the matter Hilbert space leading to few level systems destroys gauge invariance. Here I show that it is possible to generalize the concept of gauge invariance, in order to obtain cavity QED models able to satisfy this principle in a consistent and meaningful way [3,6]. The proposed generalized minimal coupling replacement can be related to the general framework of lattice gauge theory and to the so called Peierls substitution . As an application, I show how, applying this theoretical framework, it is possible to obtain a modified quantum Rabi model able to provide gauge-invariant physical results (e.g., energy levels, expectation values of observables, quantum probabilities) in any interaction regime [3,7].

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The one-fluid extended model of superflui helium II: recent results

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Abstract

I present our recent results on the one-fluid extended model. In particular, we perform the first numerical comparison between the two main existing models of superfluid helium: the two-fluid model proposed by Landau and the one-fluid extended model proposed from the extended thermodynamics. The numerical experiments in this paper regard the profiles of the so-called normal and superfluid components in 2D counterflow turbulence. We are also interested to study the influence of the boundary conditions of the velocity fields. To make progress, we also perform numerical simulations where we allow a slip velocity of the viscous component at the walls, and observe how this impacts on velocity fields and density profiles of distribution of quantized vortices.

Keyword: liquid helium; quantized vortices; counterflow channel

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Phonon-boundary scatterings and boundary conditions: Application to the heat transfer in thin nano-wires

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At nano-scale several conceptual questions still remain unanswered; some of them are sometimes fervently debated. It is however currently well-known that in nano-systems the phonons always undergo to different scattering mechanisms during the heat propagation. The particular regime of heat transfer, instead, strictly depends on the ratio between the phonon mean-free path and the characteristic size of the system, i.e., the so-called Knudsen number.

Among the aforementioned open questions, the role played by the phonon-boundary scatterings, for different values of the Knudsen number, in particular is an interesting research playground.

In this talk, therefore, it will be proposed an enhanced model of boundary conditions in order to suitably address the problem of the correct tackling of the phonon-boundary scatterings when heat is flowing in a thin nano-wire.

Theory of Quasi-Hamiltonian Dynamics for the Numerical Simulation of Equilibrium and Nonequilibrium Systems

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Since its birth with the works of Berni and Alder, Fermi, Pasta, and Ulam, numerical simulation on digital computers has been a powerful tool for scientific research, complementing theory and experiments. One of the main hurdles this method has to confront is the memory requirements imposed by many-body systems. For example, thermodynamic reservoirs require an infinite number of degrees of freedom to be instantiated. Instead, quasi-Hamiltonian dynamics can represent thermodynamic reservoirs using a few additional phase space coordinates. In this talk, we show how such a theory is formulated by employing quasi-Lie brackets, violating the Jacobi relation, and discuss its implementation for both equilibrium and nonequilibrium systems.

The Origin of Quantum Statistics: Some Insights from Quantum Cognition

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Identical systems, or *entities*, are *indistinguishable* in quantum mechanics (QM), and the *symmetrization postulate rules* the possible statistical distributions of many identical quantum entities. However, a thorough analysis on the historical development of QM attributes the origin of quantum statistics, in particular, *Bose–Einstein statistics*, to a lack of statistical independence of the micro-states of identical quantum entities. We have recently identified Bose–Einstein statistics in the combination of words in large texts, as a consequence of the *entanglement* created by the meaning carried by words when they combine in human language. Relying on this investigation, we put forward the hypothesis that entanglement, hence the lack of statistical independence, is due to a *mechanism of contextual updating*, which provides deeper reasons for the appearance of Bose–Einstein statistics in human language. However, this investigation also contributes to a better understanding of the origin of quantum mechanical statistics in physics. Finally, we provide new insights into the intrinsically random behaviour of microscopic entities that is generally assumed within classical statistical mechanics.

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Unveiling and veiling an entangled light-matter quantum state from the vacuum

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The ground state of an atom interacting with the electromagnetic field in the ultrastrong coupling regime is composed of virtual photons entangled with the atom. We propose a method to promote to real the entire photonic state, while preserving the entanglement with the atom. The process can be
reversed, and the entangled state can be restored in the vacuum. We consider a four-level atom, with two of these levels ultrastrongly coupled to a cavity mode. The process is obtained by making use of either an ideal ultrafast pulse or a more realistic multitone pulse that drives only the atom.
An experimental realization of this proposal will not only enable the investigation of the exotic phenomena of emission of particles from the vacuum, but will also prove that quantum superposition states can be extracted from the vacuum. Moreover, it will allow one to inspect the ground state in the ultrastrong coupling regime, and to generate on-demand entangled states for quantum information processing.

Roberto Stassi, Mauro Cirio, Ken Funo, Jorge Puebla, Neill Lambert, and Franco Nori, Phys. Rev. Research 5, 043095 – Published 27 October 2023

From Quantum Microstates to Macrostates: The Role of Sectorization and Factors in Wigner's Friends Problem

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We examine the shift from unitary, reversible quantum evolution, as described by the von Neumann-Everett formalism, to non-unitary, irreversible processes in quantum measurements. By analyzing quantum systems through the lens of infinite tensor products, we explore the phenomenon of nested and iterated Wigner's friend scenarios. These infinite structures challenge unitarity by introducing sectorization and factorization effects, breaking unitarity equivalence. Connections to real analysis, recursion theory, and principles from statistical physics provide further insight into the nature of this breakdown and its implications for quantum measurement theory.

Many-particle systems with contact interactions

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Quantum Hamiltonians with contact (or zero-range) interactions are useful models to analyze the behaviour of quantum systems at low energy in different contexts. In this talk we discuss recent mathematical results on the construction of such Hamiltonians for a system of $N \geq 3$ interacting bosons in dimension three as self-adjoint and lower bounded operators in the appropriate Hilbert space. We will also show the connection with a previous result obtained by Albeverio, Hoegh-Krohn and Streit in 1977 and we will discuss possible applications to the Efimov effect.
The talk is based on a series of works in collaboration with G. Basti, C. Cacciapuoti, D. Ferretti, R. Figari, D. Finco and H. Saberbaghi.

Port to GPU of a deterministic solver for confined devices

Francesco Vecil
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We show how we have ported to GPU a Boltzmann-Schroedinger-Poisson solver for DG-OSFETs, in order to reduce the time consumption of the simulations.
In particular, we describe how we have implemented a Scheduled Relaxed Jacobi (SRJ) iterative method for the solution of the sparse linear systems arising in the 2D Poisson equation.
The global speedup obtained on a Nvidia Tesla V100 with respect to a one-core simulation is around 340, and around 30 with respect to a 16-core OpenMP simulation.

Asymmetries in the brain and behaviour at the individual and population levels arising as a consequence of the minimization of energy and free energy

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Several theories have been put forward to account for the existence and maintenance in the evolution of the asymmetric organization of the brain at both individual and population levels, as documented in species ranging from *Caenorhabditis elegans* nematodes to humans. A general framework[1] is here presented where at the individual level the antisymmetric singlet configuration is shown to realize the lowest energy state of the system, whereas at the group level, the spontaneous emergence of directional asymmetry arises as a consequence of the minimization of the free energy of the system, whose stability and equilibrium is thus ensured. It is therefore suggested that the various phenomenological aspects of brain asymmetry that have been observed in biology, from, e.g., sparing of neural tissue to control of unitary motor responses, may be thought of as manifestations of the more general principle of energy and free energy minimization.

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Challenges of time-dependence in quasi-Hermitian quantum dynamics

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In an auxiliary, manifestly unphysical but mathematically maximally user-friendly Hilbert space, unitary quantum systems can be characterized by certain time-dependent operators which are manifestly non-Hermitian. Unfortunately, the implementation of the condition of time-dependence makes the description of the evolution almost prohibitively complicated.
At the same time, at least a partial reduction of the complications will be presented in the talk because it seems to be more than sufficiently compensated by the related enhancement of the flexibility and applicability of the theory. For illustration, attention will be paid to the anomalies emerging in response to small perturbations, especially near the so called exceptional point singularities. I.a., this will enable us to explain the inconsistency of certain popular but ill-defined models sampled by the imaginary cubic oscillator for which the correct physical (i.e., unitarity-supporting) inner-product metric and norm are found not to exist at all. In parallel, a consistent regularization of these traditional benchmark systems is found provided by a family of their fully consistent discrete analogues.