Bifurcation Analysis and Turing Pattern Formation in a Crimo-Taxis model with Cross-Diffusion

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Urban crime is an increasingly important issue and represents a complex phenomenon influenced by various socio-economic, cultural, and environmental factors, as poverty, unemployment, inequality, lack of opportunities, and high population density [1]. These factors often contribute to a rise in criminal activities, ranging from physical violence to property crimes and organized crime. The consequences of crime are profound, leading to fear, insecurity, and a lack of trust in public institutions. To address this, predictive models are essential to forecast areas most at risk of crime, enabling law enforcement to focus resources on vulnerable zones. This study explores two theoretical models, that was introduced in [2], aimed at simulating the spatio-temporal evolution of crime, with particular emphasis on the emergence of Turing patterns [3] induced by a cross-diffusion term, self-organizing spatial structures that may provide insights into the distribution and evolution of criminal behavior. The total population is divided into three subgroups: ordinary citizens, drug users/dealers, and law-enforcement. The interactions and spatial distributions among these group evolve over time. Inspired by epidemiological models, this approach employs reaction-diffusion and kinetic models to better understand urban crime dynamics and to explore potential intervention strategies. In this work, the positivity of the models is analyzed, ensuring the physical relevance of the population densities involved. Moreover, for both variant proposed in [2], a bifurcation analysis is performed using the pde2path algorithm [5], allowing us to identify the critical values of the diffusion coefficient for which stable Turing-type spatial patterns emerge. Also, the numerical results are obtained using an IMEX (Implicit–Explicit) finite difference scheme and confirm the theoretical predictions, showing the formation of spatial patterns that are relevant for describing the spread of crime in urban environments.

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