Beyond Saint-Venant: A critical review of physical models for lava flow simulation and the advantages of viscous-limit diffusion approaches

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The accurate simulation of lava flow emplacement is a cornerstone of volcanic hazard assessment. Historically, many numerical models have been adapted from the Saint-Venant (Shallow Water) equations, which were originally designed for low-viscosity fluids where inertia and advection dominate. In such frameworks, rheological properties and viscosity are often relegated to a source term, a mathematical treatment that can lead to numerical instabilities and computational inefficiency when applied to the high-viscosity regime typical of most basaltic and intermediate lavas.

This work provides a critical review of current simulation methodologies, highlighting the physical contradictions of using advection-centric models for flows characterized by extremely low Reynolds numbers. We propose a shift in perspective: by neglecting the advective term, which is physically negligible in slow, viscous volcanic flows, the momentum balance is simplified into a non-linear diffusion equation for flow thickness. This approach is not only more physically consistent with the rheology of lava but also numerically more robust.

To demonstrate the efficacy of this approach, we present, an new C++ code optimized for both CPU and GPU (CUDA) architectures. By leveraging the diffusion-based model, this model achieves high-performance simulations, capable of modeling a full day of emplacement in approximately a couple of minutes on standard hardware. The model's accuracy is validated through established benchmarks and then applied to real-world scenarios, specifically the 1991 and 2001 eruptions of Mt. Etna. Results show that the viscous-limit approximation provides a superior balance between computational speed and morphological accuracy, making it an ideal tool for near real-time simulations.