Fluid statics of a perfect-gas isothermal sphere with general relativity

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ABSTRACT

Our interest about self-gravitating fluids traces back to 2019 and was motivated by the will to investigate gravito-magnetism, a still rather controversial subject nowadays. For more details, we refer the reader to the introduction of the first article in our series [1–3] addressing self-gravitating fluids and to the bibliographic references [6-44] contained therein.

The main task of Ref. [1] was the determination of the fluid-static equilibrium configuration of a self-gravitating gas sphere as required initial condition needed to start the dynamics. As everybody else did before us since the times of the pioneers [4–10], we assumed the perfect-gas model (PG-m) to characterize the physical behavior of the gas sphere, reached the isothermal Lane-Emden differential equation [Eq. $(27b)_{[1]}$][1] and obtained the same peculiar results: peripheral-density spiral recurrence (Fig. $5_{[1]}$), central-density oscillation (Fig. $6_{[1]}$), gravitational-number [Eq. $(37a)_{[1]}$] upper boundedness, solution multiplicity (Figs. 7-$8_{[1]}$). Notwithstanding their abundant appearance in the astrophysical literature, these results have always struck us as somewhat odd and suspiciously unphysical and we have been wondering about the possible causes. One strongly suspectable candidate in this regard is the basic assumption of the PG-m according to which there is complete absence of molecular forces among the particles composing the gas; its representability with respect to a self-gravitating gas sphere placed in interstellar space, whose macroscopic configuration of fluid-static equilibrium is a direct and sole consequence of microscopic molecular gravitational forces, maybe is a bit too long stretch. We have to keep in mind that the equation of state

$$p = \rho RT \tag{1}$$

is the macroscopic outcome of a statistical procedure that, whether classical or quantum-mechanical, springs microscopically from a Hamiltonian formed only with kinetic-energy terms; the potential-energy terms must be switched off and this condition can be enforced only by setting the gravitational constant to the mathematically required although physically unrealistic value

$$G = 0 \ . \tag{2}$$

Therefore Eq. (1), which belongs to a model, fits only real-life circumstances in which the whole physical phenomenology attached to the gravitational constant is negligible and does not play any role; thus, compliance with such requirement calls for the implementation of Eq. (2) everywhere in the mathematical scheme built to describe the physical phenomenology, even at macroscopic level. With a view to the isothermal Lane-Emden equation [Eq. $(27b)_{[1]}$], for example, Eq. (2) cuts off the source term and deprives the gas sphere of any capability to self-gravitate. The suspicion is inevitable: PG-m and self-gravity appear to be irreconcilable concepts within the Newtonian theory of gravity.

In Ref. [2], we replaced the PG-m with the van der Waals gas model (vdWG-m) in the hope to gain physical consistency, at least qualitatively. This move repristinates the gravitational constant to its physical value, which becomes macroscopically evident through the existence of the van der Waals' constant in the equation of state that accounts for intermolecular forces [11]. We selected a couple of physically meaningful values for both that constant and the other one that accounts for molecular size [Eqs. $(53)_{[2]}$], repeated the calculation program developed in Ref. [1], and found out that the unphysical results featured by the PG-m had almost completely disappeared. There remained the question regarding multiple solutions. We felt prompted to investigate if the solutions' uniqueness we found with the selected values of the van der Waals' constants was systematic or if there could be some values of the constants in correspondence to which the PG-m's unphysical results reappeared. We dealt with such a problem in Ref. [3] and found out that multiple solutions

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[1] We maintain the convention to subscript cross-references to our series of studies [1–3] on self-gravitating fluids with the labels [1], [2] and [3], respectively; for example, Fig. $1_{[1]}$ refers to Fig. 1 in Ref. [1] and Eq. $(3)_{[2]}$ refers to Eq. (3) in Ref. [2].

do exist also for the vdWG-m but their genesis originates from reasons completely different from those of the PG-m. We provided the boundary, in the graphical plane formed with the two van der Waals' constants, demarking the separation between the zones of solutions' uniqueness and multiplicity (FIGS. 21-22$_{[3]}$).

The results of the investigations with the vdWG-m [2, 3] converted our suspicion that the PG-m is not an adequate model to represent self-gravitating fluids within the Newtonian theory of gravity into certain conviction. On the other hand, non-interacting particles can exist in the spacetime of general relativity without any conceptual limitation and their presence merely deforms the spacetime geometry; then it acquires meaning to ask what general relativity has to say about the fluid static equilibrium of isothermal spheres described with the PG-m. We tasked ourselves to find out the answer as logical component to complete our series of studies [1–3] and perhaps as final attempt to revalorize the PG-m, and we take up that challenge in this communication.

Readers belonging to the general-relativity and astrophysical communities may wonder why would one deal with a problem that has been analyzed several times and from various angles by quite a few members of their communities; the answer is simple: we looked at the problem from a fluid dynamicist's angle, followed a study pathway within the corresponding perspective and encountered a few situations that led us to form opinions and conclusions somehow different from those established in the pertinent literature of the mentioned communities. We believe worthwhile to share those differences with interested scientists.

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