Automatic Computation of Optimal Systems of Lie Subalgebras

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Lie groups of symmetries of differential equations are a fundamental tool for constructing group-invariant solutions. Since the number of subgroups (and hence invariant solutions) can be potentially infinite, it is crucial to classify subgroups to obtain an optimal system of inequivalent representatives, from which all other solutions can be derived by the group action. Because of the close correspondence between Lie groups and Lie algebras, this task is equivalent to classifying inequivalent Lie subalgebras.

We present an effective algorithm that automatically computes optimal systems of Lie subalgebras for a generic finite-dimensional Lie algebra, given via structure constants, or realized as matrices or vector fields, or specified by a basis and its nonzero Lie brackets. The algorithm is implemented in Wolfram Mathematica, and its performance is shown on meaningful, nontrivial examples, including the optimal systems of Lie subalgebras of real Lie algebras of dimensions 3 and 4, as well as the optimal systems of Lie symmetries admitted by some well-known PDEs, such as the linear heat equation, Burgers' equation, and the Korteweg–de Vries equation.

[1] L. Amata, F. Oliveri, Automatic Determination of Optimal Systems of Lie Subalgebras: the Package SymboLie, in *The Diverse World of PDEs: Algebraic and Cohomological Aspects*, *Contemporary Mathematics*, 789, 2023.

[2] L. Amata, F. Oliveri, E. Sgroi, Optimal Systems of Lie Subalgebras: A computational approach, *Journal of Geometry and Physics*, 204, 2024.

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[4] L. Amata, F. Oliveri, E. Sgroi, SymboLie: a package for determining optimal systems of Lie subalgebras, Source code available at `https://mat521.unime.it/oliveri/`.