Fully-Nonlinear Symmetry-Integrable Evolution Equations and the Schwarzian Derivative

by

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Abstract: We discuss the role of the *Schwarzian Derivative* S,

$$S := \frac{u_{3x}}{u_x} - \frac{3}{2}\frac{u_{xx}^2}{u_x^2}, \tag{0.1}$$

in the classification of fully-nonlinear and symmetry-integrable evolution equations in 1+1 dimensions. The *Schwarzian Korteweg-de Vries equation*,

$$u_t = u_x S, \tag{0.2}$$

which is invariant under the Möbius Transformation in u, is a well known example of a 3rd-order semilinear symmetry-integrable equation and serves as the starting point of our discussion. We show that fully-nonlinear and symmetry-integrable evolution equations of third and higher order appear naturally in terms of the Schwarzian derivative in a more general setting, whereby it is essential that the equations are kept invariant under certain Projective Transformations [1].

Reference:

[1] M. Euler, N. Euler and F. Oliveri, On differential equations invariant under a projective transformation group: integrability and reductions, *arXiv: Exactly Solvable and Integrable Systems* (nlin.SI), 2025. https://doi.org/10.48550/**arXiv.2505.09800**