Fuzziness Across Classical and Quantum Frameworks: Łukasiewicz versus Quantum Observables

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Abstract

Fuzziness provides an important conceptual and mathematical framework for modelling real-world phenomena characterised by uncertainty, incompleteness, or imprecision, such as complex geophysical processes including volcanic activity. Beyond classical applications, fuzzy structures have also found relevance in quantum physics and in the broader domain of quantum-like modelling. However, fuzziness alone does not exhaust the distinctive features of quantum mechanics, whose non-classical behaviour also involves contextuality and non-commutativity, going beyond mere graded uncertainty. From this perspective, a systematic comparison between fuzzy and quantum frameworks offers a rigorous methodological tool for analysing and classifying different forms of non-classicality and uncertainty, thereby clarifying both their conceptual relationships and their foundational differences.

The analysis of these relationships constitutes the focus of the present work. In particular, we show that certain fuzzy classical models can be understood as restrictions of the unsharp quantum model. We begin by investigating the connections between fuzzy quantum observables, mathematically represented by commutative Positive Operator-Valued Measures (POVMs), and Łukasiewicz (or fuzzy) observables, represented by Fuzzy Event-Valued Measures (FEVMs). We demonstrate that commutative fuzzy quantum observables admit a natural interpretation as Hilbert-space representations of Łukasiewicz observables.

Finally, we examine the relationship between Łukasiewicz observables and fuzzy quantum observables within the framework of General Probabilistic Theories (GPTs). We show that a suitable restriction of the Łukasiewicz GPT is isomorphic to a corresponding restriction of the unsharp quantum GPT. As a consequence, every Hilbert-space representation of the Łukasiewicz GPT can be identified with a specific restriction of the unsharp quantum model.