Research Days in Commutative Algebra and Combinatorics
May 23-25, 2017
Messina University

On the classification of Cohen–Macaulay binomial edge ideals

Giancarlo Rinaldo
University of Trento, Italy
giancarlo.rinaldo@unitn.it

Abstract

In 2010, binomial edge ideals were introduced in [3] and appeared independently also in [7]. Let $S = K[x_1, \ldots, x_n, y_1, \ldots, y_n]$ be the polynomial ring in $2n$ variables with coefficients in a field K. Let G be a graph on vertex set $[n]$. For each edge $\{i, j\}$ of G with $i < j$, we associate a binomial $f_{ij} = x_i y_j - x_j y_i$. The ideal J_G of S generated by $f_{ij} = x_i y_j - x_j y_i$ such that $i < j$, is called *the binomial edge ideal* of G. Any ideal generated by a set of 2-minors of a $2 \times n$-matrix of indeterminates may be viewed as the binomial edge ideal of a graph.

The classification of Cohen-Macaulay binomial edge ideals in terms of the underlying graphs is still widely open and, as in the case of monomial edge ideals introduced in [9], it seems rather hopeless to give a full classification. In [6], [8], [4], [5], [1] and [2], the authors considered the Cohen-Macaulay property of these graphs.

The aim of this talk, after giving a brief survey on the subject, is to extend the results of [4] where unicyclic graphs have been studied. In particular we give a classification Cohen-Macaulay and unmixed binomial edge ideals J_G when G is a cactus graph. Thanks to these results we are able to classify the Cohen-Macaulay and unmixed binomial edge ideals J_G when G is a bicyclic graph (see [5]).

References

[1] A. Banerjee, L. Nunez-Betancourt, *Graph Connectivity and Binomial edge ideals*, Proceed. of the Am. Math. Soc., Vol. 145 (2), 2017, pp. 487–499.

[2] D. Bolognini, A. Macchia, F. Strazzanti, *Binomial edge ideals of bipartite graphs*, arXiv:1704.00152v1, 2017, pp. 1–29.

[3] J. Herzog, T. Hibi, F. Hreinsdottir, T. Kahle, J. Rauh, *Binomial edge ideals and conditional independence statements*, Advances in Applied Mathematics, volume 45, issue 3, 2010.

[4] G. Rinaldo, *Cohen-Macaulay binomial edge ideals of small deviation*, Bull. Math. Soc. Sci. Math. Roumanie Tome 56(104) No. 4, 2013, 497–503.

[5] G. Rinaldo, *Cohen-Macaulay binomial edge ideals of cactus graphs*, arXiv:1704.07106, 2017, pp. 1–10.

[6] V. Ene, J. Herzog, T. Hibi, *Cohen-Macaulay binomial edge ideals*, Nagoya Math. J. Volume 204 (2011), 57–68.

[7] M. Ohtani, *Graphs and ideals generated by some* 2*-minors*, Comm. Algebra (39), 2011, pp. 905–917.

[8] A. Rauf, G. Rinaldo, *Construction of Cohen-Macaulay binomial edge ideals*, Comm. Algebra., Taylor and Francis, ISSN: 1532-4125, 2014, pp. 238-252

[9] R.H. Villarreal, *Cohen-Macaulay graphs*, Manuscripta Math. **66**, 1990, pp. 277–293.