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On the trace of the canonical module and nearly Gorenstein rings

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Abstract

According to a famous paper of Bass, Gorenstein rings are ubiquitous. Apart from this, they admit beautiful symmetry properties: self dual resolutions, symmetric h-vectors, and, by a theorem of Kunz, among the numerical semigroup rings they are those with symmetric semigroup. There have been several nice ideas to define properties of rings which are sightly weaker than that of being Gorenstein. Barucci and Fröberg, for example, defined almost symmetric numerical semigroups and Goto, Takahashi and Taniguchi generalized this concept and introduced almost Gorenstein rings. Since then, almost Gorenstein rings have been studied in numerous papers.

In a recent paper, together with Stamat, we considered the trace of the canonical module ω_R of a local Cohen-Macaulay ring $(R, \mathfrak{m})$. The significance of this trace is that it describes the non-Gorenstein locus of R. Thus R is Gorenstein if and only if $\operatorname{tr}(\omega_R) = R$. If the trace of ω_R comes very close to R, namely if $\mathfrak{m} \subseteq \operatorname{tr}(\omega_R)$, we call R nearly Gorenstein.

In this lecture we discuss nearly Gorenstein rings for Segre products and Veronese rings. The theory will be applied to give a full classification of all Hibi rings which are nearly Gorenstein in terms of the underlying poset. .